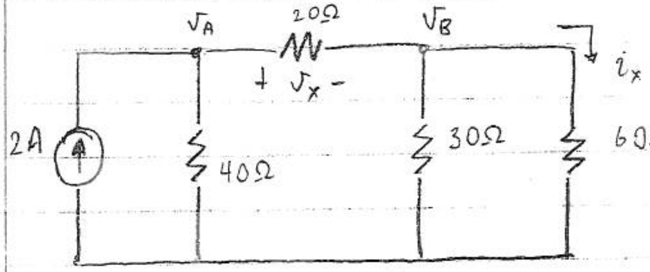
MAE 140 Fa 07 HW#3 3/1, 3, 4, 5, 8, 11, 13, 19, 71, 75

3/1



(a) KCL @ A

$$v_A\left(\frac{1}{40}\right) + (v_A - v_B)\left(\frac{1}{20}\right) = 2$$

$$\Rightarrow \quad v_A\left(\frac{1}{40}+\frac{1}{20}\right) + v_B\left(-\frac{1}{20}\right) = 2 \quad //$$

KCL@B

$$(v_B - v_A)\left(\frac{1}{20}\right) + v_B\left(\frac{1}{30}\right) + v_B\left(\frac{1}{60}\right) = 0$$

$$v_A\left(-\frac{1}{20}\right) + v_B\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right) = 0 \quad //$$

$$\boxed{\begin{bmatrix} 3/40 & -1/20 \\ -1/20 & 1/10 \end{bmatrix}\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}}$$

(b) from (a) (using a calculator)

$v_A = 40$ V

$v_B = 20$ V

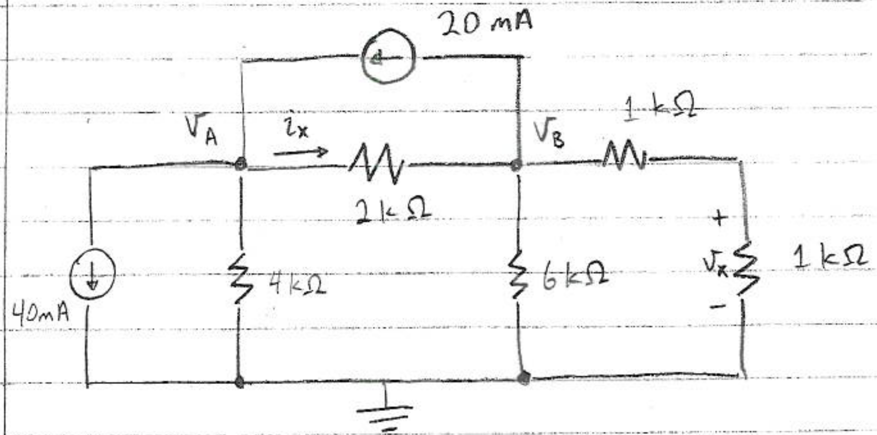
$v_x = v_A - v_B$

$\boxed{v_x = 20\text{V}}$

$i_x = \frac{v_B}{60}$

$\boxed{i_x = 1/3 \text{ A}}$

3/3



(a) KCL @ A :
KCL @ B :

$$\begin{bmatrix} \frac{1}{4} + \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{6} + \frac{1}{1+1} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 20 - 40 \\ -20 \end{pmatrix}$$

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{7}{6} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} -20 \\ -20 \end{pmatrix}$$

(b) $V_A = -53\frac{1}{3}$ V
$V_B = -40$ V

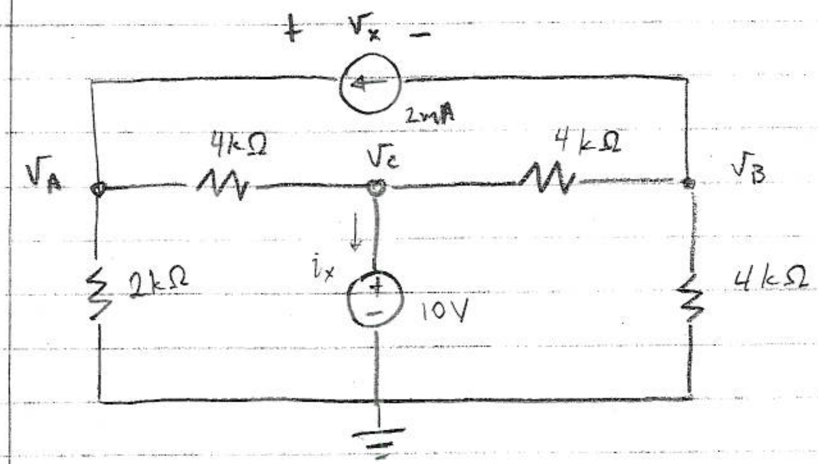
$V_x = V_B \frac{1}{1+1}$ (voltage divider)

$\boxed{V_x = -20\text{ V}}$

$i_x = \frac{V_A - V_B}{2}$ mA

$\boxed{i_x = -6\frac{2}{3}\text{ mA}}$

3/4



Note $v_c = 10$ V

KCL @ A

$$v_A\left(\frac{1}{2}+\frac{1}{4}\right) = 2 + \frac{v_c}{4}$$

$$\frac{3}{4}v_A = \frac{9}{2}$$

$\therefore v_A = 6$ V

KCL @ B

$$v_B\left(\frac{1}{4}+\frac{1}{4}\right) = -2 + \frac{v_c}{4}$$

$$\frac{v_B}{2} = \frac{1}{2}$$

$\therefore v_B = 1$ V

$$v_x = v_A - v_B$$

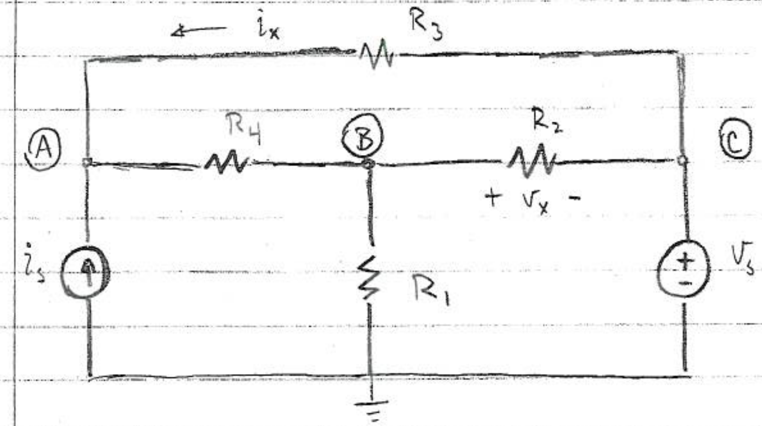
$$\boxed{v_x = 5\text{ V}}$$

$$i_x = \frac{v_A - v_c}{4} + \frac{v_B - v_c}{4}$$

$$= \frac{1}{4}(6 - 10 + 1 - 10)$$

$$\boxed{i_x = -\frac{13}{4}\text{ A}}$$

3/5



a) Note that $V_C = V_S$

KCL A / KCL B

$$\begin{bmatrix} 1/R_4 + 1/R_3 & -1/R_4 \\ -1/R_4 & 1/R_4 + 1/R_1 + 1/R_2 \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} i_s + V_s/R_3 \\ V_s/R_3 \end{pmatrix}$$

b) $R_1 = R_2 = R_3 = R_4 = 10 \text{ k}\Omega$

$V_s = 20 \text{ V}, \quad i_s = 2 \text{mA}$

$\Rightarrow V_A = 28 \text{ V}$

$V_B = 16 \text{ V}$

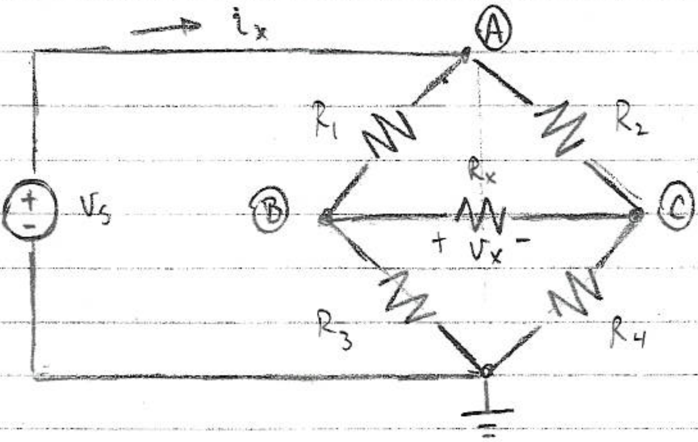
$$i_x = \frac{V_s - V_A}{R_3}$$

$\boxed{i_x = -.8 \text{ mA}}$

$V_x = V_B - V_s$

$\boxed{V_x = -4 \text{ V}}$

3/8



(a) Note $v_A = v_s$

KCL @ B
KCL @ C

$$\begin{bmatrix} 1/R_1 + 1/R_x + 1/R_3 & -1/R_x \\ -1/R_x & 1/R_2 + 1/R_x + 1/R_4 \end{bmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} v_s/R_1 \\ v_s/R_2 \end{pmatrix}$$

(b) $R_1 = R_4 = 10^3\ \Omega$

$R_2 = R_3 = 250\ \Omega$

$R_x = 500\ \Omega$

$v_s = 15\ V$

$v_B = 5\ V$

$v_C = 10\ V$

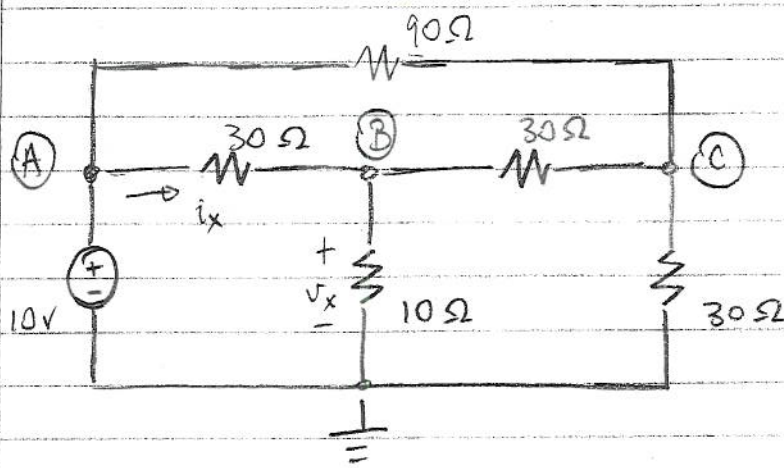
$$i_x = \frac{v_s - v_B}{r_1} + \frac{v_s - v_C}{R_2}$$

$i_x = 30\ mA$

$v_x = v_B - v_C$

$v_x = -5\ V$

3/11



Note $V_A = 10\text{ V}$

KCL B
KCL C

$$\begin{bmatrix} \frac{1}{30}+\frac{1}{10}+\frac{1}{30} & -\frac{1}{30} \\ -\frac{1}{30} & \frac{1}{30}+\frac{1}{30}+\frac{1}{90} \end{bmatrix} \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 10/30 \\ 10/90 \end{pmatrix}$$

$V_B = 2.5\text{ V}$

$V_C = 2.5\text{ V}$

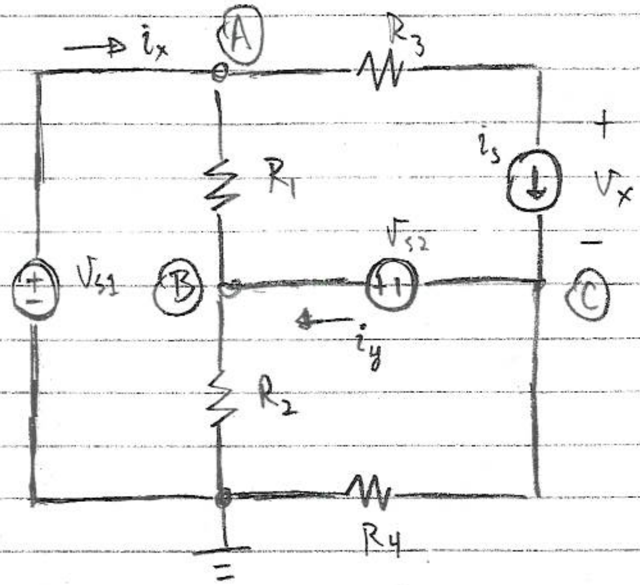
$V_x = V_B$

$\boxed{V_x = 2.5\text{V}}$

$i_x = \frac{10 - V_B}{30}$

$\boxed{i_x = 250\text{ mA}}$

3/13



(a) note that
$$\begin{cases} V_C + V_{S2} = V_B \iff V_C = V_B - V_{S2} \\ V_A = V_{S1} \end{cases}$$
//

KCL B: $V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = i_y + \frac{V_{S1}}{R_1}$ (1)

KCL C: $\frac{V_B - V_{S2}}{R_4} = -i_y + i_s$ (2)

Combine (1) and (2)

$$V_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_B}{R_4} - \frac{V_{S2}}{R_4} = \frac{V_{S1}}{R_1} + i_s$$

$$V_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) = \frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_4} + i_s$$ //

(b) $R_1 = R_2 = 10\,k\Omega$, $R_3 = 2\,k\Omega$, $R_4 = 1\,k\Omega$, $i_s = 2.5\,mA$, $V_{S1} = 12\,V$, $V_{S2} = .5\,V$

$\Rightarrow V_B = 3.5\ V$

$V_C = 3\,V$

$V_X = V_A - i_s R_3 - V_C$

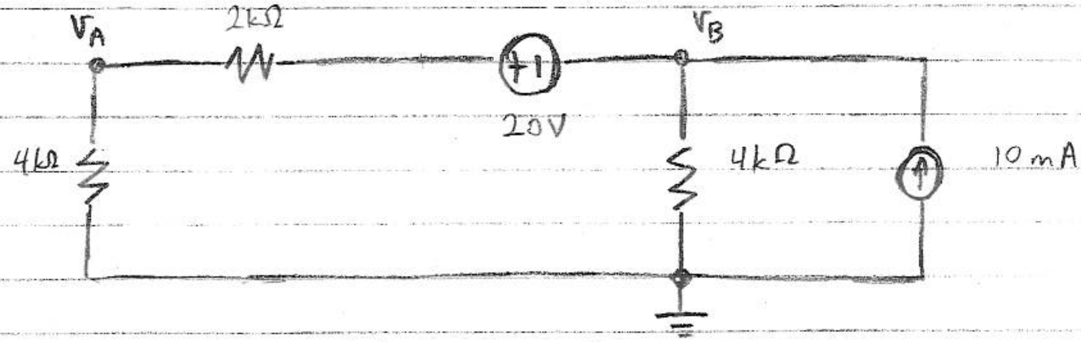
$\boxed{V_x = 4\,V}$

$i_x = \frac{V_{S1} - V_B}{R_1} + i_s$

$\boxed{i_x = 3.35\ mA}$

$\boxed{P = V_{S1}\, i_x = 40.2\,W \text{ generated}}$

3/19



KCL @ A: $V_A\left(\frac{1}{4}\right) + (V_A - 20 - V_B)\left(\frac{1}{2}\right) = 0$

$\Rightarrow \quad V_A\left(\frac{3}{4}\right) + V_B\left(-\frac{1}{2}\right) = 10$

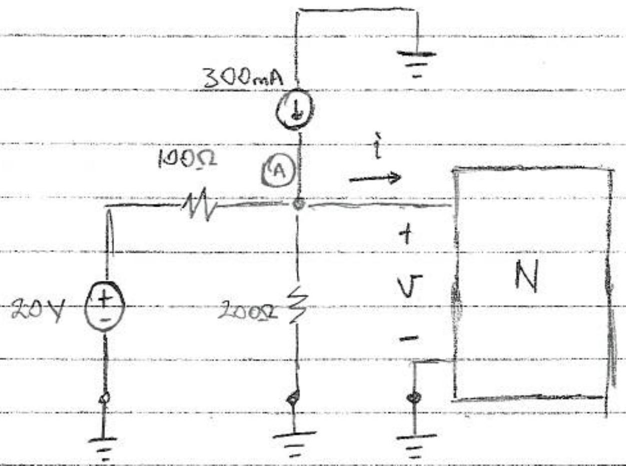
KCL @ B: $V_B\left(\frac{1}{4}\right) + (V_B + 20 - V_A)\left(\frac{1}{2}\right) = 10$

$\Rightarrow \quad V_A\left(-\frac{1}{2}\right) + V_B\left(\frac{3}{4}\right) = -10 + 10$

$$\begin{bmatrix} 3/4 & -1/2 \\ -1/2 & 3/4 \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

$V_A = 24$ V
$V_B = 16$ V

3/71



$i = (5 \times 10^{-3}\, v - .5)$ A * note that $v_A = v$

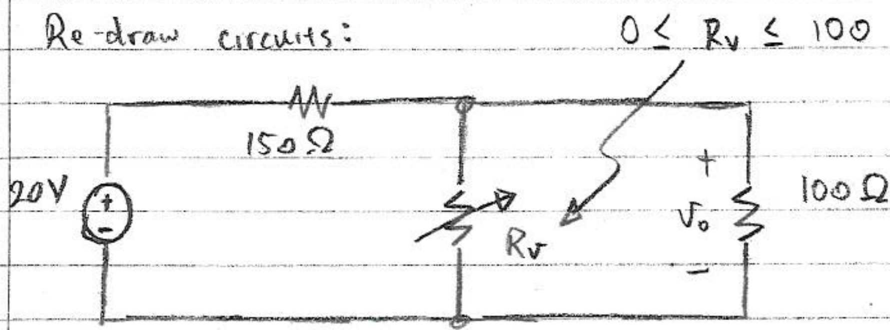
KCL @ A:

$$\frac{1}{100}(v_A - 20) + \frac{v_A}{200} + 5 \times 10^{-3}\, v_A = .3 + .5$$

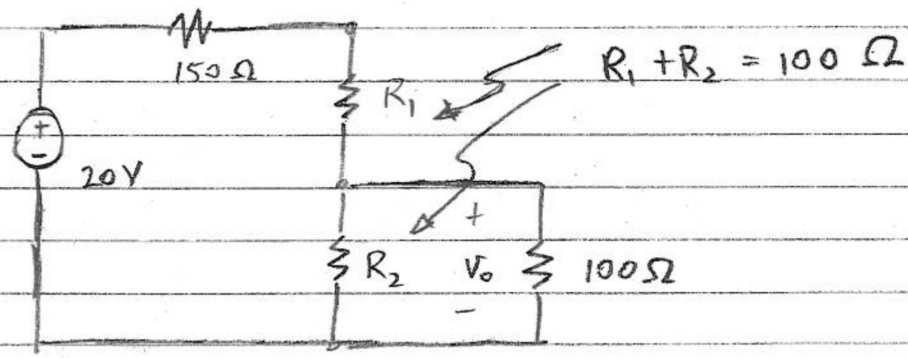
$$\frac{v_A}{50} = 1$$

$$\Rightarrow \boxed{v_A = 50 \text{ V}}$$

3.75 Re-draw circuits:



Circuit 1



Circuit 2

For circuit 1

$$V_o = 20 \times \left(\frac{\frac{100 R_v}{100 + R_v}}{150 + \frac{100 R_v}{100 + R_v}} \right)$$

$$V_o = 20 \times \frac{100 R_v}{15{,}000 + 250 R_v}$$

For circuit 2

$$V_o = 20 \cdot \frac{100 R_2}{(150 + R_1)(100 + R_2) + 100 R_2}$$

where $R_1 + R_2 = 100$

$$V_o = 20 \times \frac{100 R_2}{25{,}000 + 250 R_2 - R_2^2}$$

From the voltage division formulae for each circuit, one can see that for a desired V_o close to zero, V_o is more sensitive to changes in R_v than R_2, ie controlling the first circuit is more difficult when the desired output is close to zero. Physically, this is similar to turning a volume knob very little and suddenly having your eardrums blasted out. One can also argue that cct 2 draws less power.