14. 8

From figure . Ex. 14.7,

$$C = 10^{-8}\ m/\text{cycle}$$

$$m = \frac{10^{-4} - 10^{-6}}{52 - 14} = 2.61 \times 10^{-6} \frac{m}{\text{cycle} \cdot MPa\sqrt{m}}$$

$$\Delta K = Y \Delta\sigma \sqrt{\pi a}$$

$$\Delta\sigma = \frac{\Delta K}{Y\sqrt{\pi a}} = \frac{10 \times 10^{6}\ Pa \cdot m^{1/2}}{1.12 \times \sqrt{\pi \times 1 \times 10^{-3}}} = 1.59 \times 10^{8}\ Pa = 159\ MPa$$

$$\Delta N = \frac{a_f^{(1-\frac{m}{2})} - a_i^{(1-\frac{m}{2})}}{C(Y\Delta\sigma\sqrt{\pi})^m (1 - \frac{m}{2})}$$

$$a_f^{1-\frac{m}{2}} = C\,\Delta N\,(Y\Delta\sigma\sqrt{\pi})^m (1 - \frac{m}{2}) + a_i^{(1-\frac{m}{2})}$$

$$= 10^{-8} (m/\text{cycle}) \times 10^{5}\ \text{cycles} \times (1.12 \times 159 \times 10^{6} \sqrt{\pi})^{2.61 \times 10^{-6}}$$
$$(1 - \frac{2.61 \times 10^{-6}}{2}) + (0.2 \times 10^{-3})^{(1 - \frac{2.61 \times 10^{-6}}{2})}$$

$$a_f^{1-\frac{m}{2}} = 1.2 \times 10^{-3}\ m$$

$$a_f = (1.2 \times 10^{-3})^{1/\left(1 - \frac{2.61 \times 10^{-6}}{2}\right)} = 1.20 \times 10^{-3}\ m$$

14.9

$$\frac{da}{dN} = C \cdot \Delta K^2$$

$$\Delta K = 2\alpha \, \Delta\sigma \sqrt{\pi a}$$

Show that $C = \dfrac{1}{4N\Delta\sigma^2 \alpha^2 \pi} \ln \dfrac{a_f}{a_o}$

$$\frac{da}{dN} = C \times (4\alpha^2 \Delta\sigma^2)\pi a$$

$$\int_{a_o}^{a_f} \frac{da}{a} = 4C\alpha^2 \Delta\sigma^2 \pi \int_0^N dN$$

$$\ln\left(\frac{a_f}{a_o}\right) = 4C\alpha^2 \Delta\sigma^2 \pi \quad N$$

$$C = \frac{1}{4N\alpha^2 \Delta\sigma^2 \pi} \ln\left(\frac{a_f}{a_o}\right)$$

14.10

Fatigue crack propagation

$$\frac{da}{dN} = 0.5 \times 10^{6} \, \Delta K^{3.5}$$

Given $B = 10\,mm$ $\qquad P = 200N$

$W = 50mm$

$2a = 10mm$

$$\Delta K = f\left(\frac{a}{W}\right) \frac{P}{B\sqrt{W}}$$

$$f\left(\frac{a}{W}\right) = \frac{2 + \frac{a}{W}}{\left(1 - \frac{a}{W}\right)^{3/2}} \left[0.866 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^{2} + 14.72\left(\frac{a}{W}\right)^{3} - 5.60\left(\frac{a}{W}\right)^{4} \right]$$

$$f\left(\frac{5}{50}\right) = f\left(\frac{1}{10}\right)$$

$$= \frac{2 + 0.1}{(1 - 0.1)^{3/2}} \left[0.866 + 4.64(0.1) - 13.32(0.1)^{2} + 14.72(0.1)^{3} - 5.60(0.1)^{4} \right]$$

$$= 2.978$$

$$\Delta K = 2.978 \times \frac{200\,N}{10 \times 10^{-3} \times \sqrt{50 \times 10^{-3}}} = 2.66 \times 10^{5} \, Pa \cdot m^{1/2}$$

14.19

Stress range = 310 MPa ; $N = 10^4$ cycles

stress range = 230 MPa ; $N = 10^7$ cycles

The material obeys Baquin's law,

$$\Delta\sigma N_f^a = C$$

Apply 2 conditions, $(310 \text{ MPa})(10^4 \text{ cycles})^a = (230 \text{ MPa})(10^7 \text{ cycles})^a$

$$\Rightarrow \frac{310 \text{ MPa}}{230 \text{ MPa}} = \left(\frac{10^7 \text{ cycles}}{10^4 \text{ cycles}}\right)^a$$

$$\Rightarrow 1.35 = 10^{3a}$$

$$\Rightarrow \log(1.35) = 3a$$

$$\Rightarrow a = 4.34 \times 10^{-2}$$

$$C = (310 \text{ MPa})(10^4 \text{ Cycles})^{4.34\times10^{-2}}$$

$$\Rightarrow C = 4.62 \times 10^2 \text{ MPa}$$

if $\Delta\sigma = 180$ MPa $\Rightarrow N_f^a = \frac{C}{\Delta\sigma} = \frac{4.62\times10^2 \text{ MPa}}{180 \text{ MPa}} = 2.57$

$$N_f = (2.57)^{\frac{1}{4.34\times10^{-2}}}$$

$$N_f = 2.79 \times 10^9 \text{ cycles}$$

we have $400 \frac{\text{rev}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{16 \text{ hrs}}{1 \text{ day}} = 3.84 \times 10^5 \frac{\text{cycles}}{\text{day}}$.

days to failure $= 2.79 \times 10^9 \text{ cycles} \times \frac{1 \text{ day}}{3.84\times10^5 \text{ Cycles}}$

$$= 7.27 \times 10^3 \text{ days}$$

14.8 For the 7075-T6 alloy in Exercise 14.7, determine the length of a crack after 105 cycles if the initial crack size was equal to 0.2 mm and the cyclic loading was such that, at the onset of fatigue, *ΔK* = 10 MPa m1/2.

From figure Ex. 14.7,

$$c = 10^{-8} m / cycle$$

$$m = \frac{10^{-4} - 10^{-6}}{52 - 14} = 2.61 * 10^{-6} \frac{m}{cycles * MPa\sqrt{m}}$$

$$\Delta k = Y \Delta\sigma \sqrt{\pi a}$$

$$\Delta\sigma = \frac{\Delta k}{Y\sqrt{\pi a}} = \frac{10 * 10^{6} Pa * m^{1/2}}{1.12 * \sqrt{\pi * 1 * 10^{-3} m}} = 1.59 * 10^{8} Pa = 159 MPa$$

$$\Delta N = \frac{a_f^{1-\frac{m}{2}} - a_i^{1-\frac{m}{2}}}{c(Y\Delta\sigma\sqrt{\pi})^m (1 - \frac{m}{2})}$$

$$a_f^{1-\frac{m}{2}} = c\Delta N (Y\Delta\sigma\sqrt{\pi})^m (1 - \frac{m}{2}) + a_i^{1-\frac{m}{2}}$$

$$a_f^{1-\frac{m}{2}} = 10^{-8}(m / cycles) * 10^{5} cycles * (1.12 * 159 * 10^{6} Pa\sqrt{\pi})^{2.61*10^{-6}} (1 - \frac{2.61 * 10^{-6}}{2}) + (0.2 * 10^{-3})^{(1-\frac{2.61*10^{-6}}{2})}$$

$$a_f = (1.2 * 10^{-3})^{\frac{1}{1-\frac{2.61*10^{-6}}{2}}} = 1.20 * 10^{-3} m$$