1

prob 1	40
prob 2	15
prob 3	25
total	80

$$\frac{dc}{dt} = \frac{r}{100} c$$

$$\frac{dc}{c} = \frac{r}{100} dt$$

$$\int_{c_0}^{c} \frac{dc}{c} = \int_{t_0}^{t} \frac{r}{100} dt$$

$$\ln c \Big|_{c_0}^{c} = \frac{r}{100} t \Big|_{t}^{t_0}$$

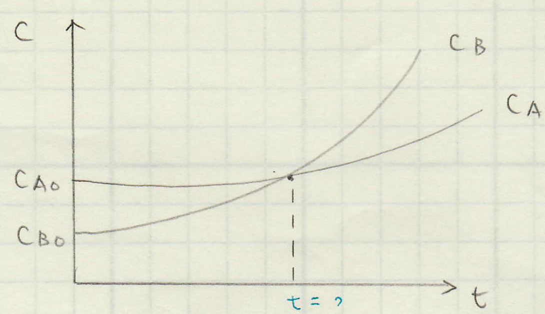
$$\boxed{c = c_0 \exp \frac{r}{100}(t - t_0)}$$

	Commodity A	Commodity B	
current consumption rate per year	C_{A0}	C_{B0}	$C_{A0} > C_{B0}$
consumption rate after each year	r_A	r_B	$r_B > r_A$

ⓐ Derive an eqn. for the time before the annual consumption of B exceeds that of A.

For A: $C_A = C_{A0} \exp\left(\frac{r_A}{100} t\right)$; with $t_0 = 0$ (present day)

$C_B = C_{B0} \exp\left(\frac{r_B}{100} t\right)$



ⓐ $C_A = C_B \Rightarrow$ Find t ?

#1 ⓐ (continue)

$$C_{A0} \exp\left(\frac{r_A}{100} t\right) = C_{B0} \exp\left(\frac{r_B}{100} t\right)$$

$$\frac{C_{A0}}{C_{B0}} = \exp\left(\frac{r_B - r_A}{100} t\right)$$

$$\ln\left(\frac{C_{A0}}{C_{B0}}\right) = \frac{r_B - r_A}{100} t$$

$$\boxed{t = \frac{100 \ln\left(\frac{C_{A0}}{C_{B0}}\right)}{r_B - r_A}}$$

+10

\# 1

(b) Find doubling times ?

$$t_D = \ln 2 \left(\frac{100}{r} \right) \simeq \frac{70}{r}$$

1) Iron & steel = $r = 3$

$$t_D = \ln 2 \left(\frac{100}{3} \right) \simeq \frac{70}{3} \simeq 23.3 \text{ years}$$ +5

2) Aluminum = $r = 4$

$$t_D = \ln 2 \left(\frac{100}{4} \right) \simeq \frac{70}{4} \simeq 17.3 \text{ years}$$ +5

3) polymer : $r = 5$

$$t_D = \ln 2 \left(\frac{100}{5} \right) \simeq \frac{70}{5} \simeq 14.1 \text{ years}$$ +5

\# 1

(c) Find the # of years where 1) aluminum and 2) polymers would exceed steel in consumption.

from part (a) we found time t where commodity B exceeds A.

$$t = \frac{100 \ln \left(\frac{C_{A0}}{C_{B0}} \right)}{r_B - r_A}$$

For steel : $C_{S0} = 6 \times 10^8$ tonnes , $r_S = 3\ \%$

For Al : $C_{A0} = 5 \times 10^7$ tonnes , $r_A = 4\ \%$

For polymers : $C_{P0} = 1.8 \times 10^8$ tonnes , $r_p = 5\ \%$

1 ⓒ

$$t_1 = \frac{100 \ln\left(\frac{6\times10^8}{5\times10^7}\right)}{4-3} = \underline{\underline{248.5 \text{ years}}} \quad +5$$

$$t_2 = \frac{100 \ln\left(\frac{6\times10^8}{1.8\times10^8}\right)}{5-3} = \underline{\underline{60.2 \text{ years}}} \quad +5$$

yes! Both of the cases are probable. +5

2 ⓐ p. 22 – 23 in book +5

ⓑ $r = 3\%$ $12\% \Rightarrow$ lead is used

$$C = C_0 \exp\left\{\frac{r}{100}(t - t_0)\right\}$$

$C_0 = 100\%$

$C = (100 - 12)\% = 88\%$

$$t = \frac{100}{3} \ln\left(\frac{1}{0.88}\right)$$

$t = 4.26$ years +10

3

ⓐ Exponential growth of a material is when the material is being consumed at a rate of consumption which is proportional to the current consumption rate.

+5

ⓑ

$C = C_0 + rQ$ @ $t = 0$

$C = C_0 + r\frac{1}{2}Q$ @ $t = t_{1/2}$

~~+5~~

$C = C_0 \exp\left\{\frac{r}{100}(t - t_0)\right\}$

$Q = \int C\,dt$ ~~+10~~ +15

find $t_{1/2}$ when $Q = \frac{1}{2}Q$

$$\frac{1}{2}Q = \int_0^{t_{1/2}} C_0 e^{\left\{\frac{r}{100}(t-0)\right\}} dt$$

$$\frac{1}{2}Q = \frac{100}{r} C_0 e^{\left\{\frac{rt}{100}\right\}}\Big|_0^{t_{1/2}}$$

$$\frac{1}{2}Q = \frac{100}{r} C_0 e^{\left\{\frac{r}{100}(t_{1/2})\right\}} - \frac{100\,C_0}{r}$$

$$\frac{rQ}{200C_0} = e^{\left\{\frac{r}{100}t_{1/2}\right\}} - 1$$

$$\frac{r}{100}t_{1/2} = \ln\left\{\frac{rQ}{200C_0} + 1\right\}$$

$$\boxed{t_{1/2} = \frac{100}{r}\ln\left\{\frac{rQ}{200C_0} + 1\right\}}$$ ~~+5~~

ⓒ p. 22 – 23 in book.

+5