17.1 A component is made of a steel for which $K_C = 54\ MN\,m^{-3/2}$. Non-destructive testing by ultrasonic methods shows that the component contains cracks of up to $2a = 0.2\,mm$ in length. Laboratory tests show that the crack-growth rate under cyclic loading is given by

$$\frac{da}{dN} = A(\Delta K)^4$$

where $A = 4\times10^{-13}\ (MN\,m^{-2})^{-4}\ m^{-1}$. The component is subjected to an alternating stress of range.

$$\Delta\sigma = 180\ MN\,m^{-2}$$

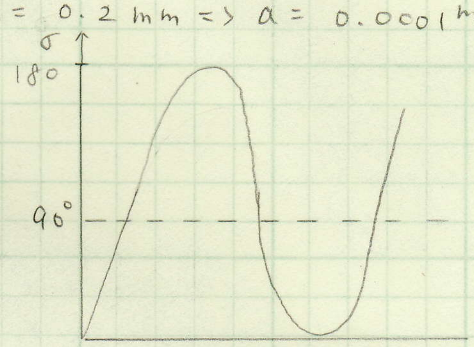
about a mean tensile stress of $\Delta\sigma/2$. Given that $\Delta K = \Delta\sigma\sqrt{\pi a}$, calculate the number of cycles to failure.

$$\frac{da}{dN} = A(\Delta K)^4$$

$K_C = 54\ MN\,m^{-3/2}$

$A = 4\times10^{-13}\ (MN\,m^{-2})^{-4}\ m^{-1}$

$2a = 0.2\,mm \Rightarrow a = 0.0001\,m$



$$dN = \frac{da}{A(\Delta K)^4}$$

$$dN = \frac{da}{A(\Delta\sigma\sqrt{\pi a})^4}$$

$$N = \frac{1}{A\Delta\sigma^4\pi^2}\int\frac{1}{a^2}\,da$$

$$N = \frac{1}{A\Delta\sigma^4\pi^2}\left.\frac{-1}{a}\right|_{a_0}^{a_f}$$

$$\left(a_f = \left(\frac{K}{\sigma\sqrt{\pi}}\right)^2 = 0.029\right)$$

$$N = \left.\frac{-da}{4\times10^{-13}(180)^4(\pi^2)\,a}\right|_{0.0001}^{0.029} = 2.4\times10^6 \text{ cycles}$$

MAE 160 | HW #7 | Solutions

17.2 An aluminum alloy for an airframe component was tested in the laboratory under an applied stress which varied sinusoidally with time about a mean stress of zero. The alloy failed under a stress range, $\Delta\sigma$, of 280 $MN\,m^{-2}$ after 10^5 cycles; under a range of 200 $MN\,m^{-2}$, the alloy failed after 10^7 cycles. Assuming that the fatigue behaviour of the alloy can be represented by

$$\Delta\sigma (N_f)^a = C$$

where a and C are material constants, find the number of cycles to failure, N_f, for a component subjected to a stress range of 150 $MN\,m^{-2}$.

$$\Delta\sigma (N_f)^a = C$$

Given $\Delta\sigma_1 = 280\ MN\,m^{-2}$ $\quad N_{f1} = 10^5$ cycles

$\Delta\sigma_2 = 200\ MN\,m^{-2}$ $\quad N_{f2} = 10^7$ cycles

$$\Delta\sigma_1 (N_{f1})^a = C \quad \text{—— (1)}$$

$$\Delta\sigma_2 (N_{f2})^a = C \quad \text{—— (2)}$$

(1) ÷ (2) $\Rightarrow$ $\dfrac{\Delta\sigma_1}{\Delta\sigma_2} \times \left(\dfrac{N_{f1}}{N_{f2}}\right)^a = 1$

$$\ln\left(\frac{\Delta\sigma_1}{\Delta\sigma_2}\right) + a \ln\left(\frac{N_{f1}}{N_{f2}}\right) = \ln(1) = 0$$

$$a = \ln\left(\frac{\Delta\sigma_2}{\Delta\sigma_1}\right) \times \ln\left(\frac{N_{f2}}{N_{f1}}\right) = \ln\left(\frac{200}{280}\right) \times \ln\left(\frac{10^7}{10^5}\right)$$

$$a = 0.073$$

$$C = \Delta\sigma (N_f)^{0.073} = 650\ MN\,m^{-2}$$

$\Big\}\ \Delta\sigma (N_f)^{0.073} = 650$

if $\Delta\sigma = 150\ MN\,m^{-2}$

$$a \ln N_f = \ln C - \ln \Delta\sigma = \ln\left(\frac{C}{\Delta\sigma}\right)$$

$$N_f = e^{\frac{\ln\left(\frac{C}{\Delta\sigma}\right)}{a}} = 5.3 \times 10^8 \text{ cycles}$$

17.3 When a fast-breeder reactor is shut down quickly, the temperature of the surface of a number of components drops from 600°c to 400°c in less than a second. The components are made of a stainless steel, and have a thick section, the bulk of which remains at the higher temperature for several seconds. The low-cycle fatigue life of the steel is described by

$$N_f^{1/2} \Delta \epsilon^{pl} = 0.2$$

where N_f is the number of cycles to failure and $\Delta \epsilon^{pl}$ is the plastic strain range. Estimate the number of fast shut-downs the reactor can sustain before serious cracking or failure will occur. (The thermal expansion coefficient of stainless steel is $1.2 \times 10^{-5}\ k^{-1}$; the yield stain at 400°c is 0.4×10^{-3})

$$\Delta \epsilon_y = 0.4 \times 10^{-3}$$

$$\Delta \epsilon^{pl} = \alpha (T_2 - T_1) - \Delta \epsilon_y$$

$$= 1.2 \times 10^{-5}\ k^{-1} (873 - 673)\ k - 0.4 \times 10^{-3}$$

$$= 2 \times 10^{-3}$$

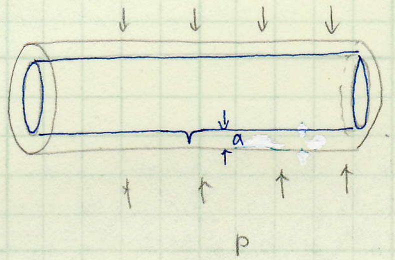
$$N_f^{1/2} \Delta \epsilon^{pl} = 0.2$$

$$N_f = \left(\frac{0.2}{\Delta \epsilon^{pl}}\right)^2 = \left(\frac{0.2}{2 \times 10^{-3}}\right)^2 = 10^4$$ shut downs before failure / cracking will occur.

17.4

(a)

Cylindrical steel pressure vessel



$D = 7.5$ m

$t = 40$ mm

$p = 5.1\ MNm^{-2}$

$K_C = 200\ MNm^{-3/2}$

$K = \sigma\sqrt{\pi a}$ a – the length of the edge – crack

σ – hoop stress

$$\sigma = \frac{pr}{t} = \frac{(5.1\ MNm^{-2})(7.5/2)}{40\times10^{-3}} = 478.3\ MNm^{-2}$$

$K_c = \sigma\sqrt{\pi a_c}$ a_c – critical length

$$a_c = \frac{K_c^2}{\sigma^2\pi} = \frac{(200\ MNm^{-2})^2}{(478.3\ MNm^{-2})^2\pi} = 0.056\ m$$

$a_c > t \Rightarrow$ failure will occur (will leak ☹)

17.4

(b) During service the growth of a flaw by fatigue is given by

$$\frac{da}{dN} = A(\Delta K)^4$$

Where $A = 2.44 \times 10^{-14} (MNm^{-2})^{-4} m^{-1}$. Find minimum pressure to which the vessel must be subjected in a proof test to guarantee against failure in service in less than 3000 loading cycles from zero to full load and back.

$$\Delta K = \Delta\sigma \sqrt{\pi a}$$

$$\Delta\sigma = \frac{\Delta pr}{t} = \frac{pr}{t} \Rightarrow \Delta K = \frac{pr}{t}\sqrt{\pi a}$$

$$\frac{da}{dN} = A(\Delta K)^4$$

$$\int_0^{N_f = 3000} dN = \int_{a_i}^{a_f} \frac{da}{A(\Delta K)^4} = \int_{a_i}^{a_f} \frac{da}{A(\Delta\sigma\sqrt{\pi a})^4}$$

$$N \Big|_0^{3000} = \frac{-1}{A(\Delta\sigma)\pi^2} \frac{1}{a} \Big|_{a_i}^{a_f = 0.040}$$

$$(3000 - 0) = \frac{1}{2.44 \times 10^{-14} \times 478.3 \times \pi^2} \times \left(\frac{1}{a_i} - \frac{1}{0.040}\right)$$

$$\frac{1}{a_i} = 63$$

$$a_i = 0.0160 \text{ m}$$

$$p = \frac{\Delta K t}{r\sqrt{\pi a_i}} = \frac{(200 \times 10^6)(0.04)}{\frac{7.5}{2} \times \sqrt{\pi \times 0.0160}} = 9.5 \text{ MNm}^{-2}$$