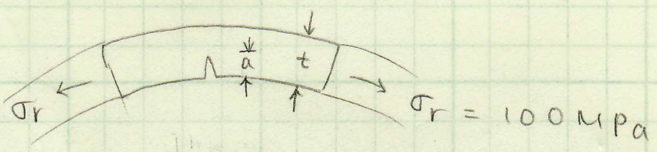
MAE 160 | HW # 6 | Soln.

15.1 A cylindrical pressure vessel is rolled up from a flat steel plate 10 mm thick, and the edges are welded together to produce a longitudinal welded seam. Calculate the stress intensity factor K at a hoop stress of $100\ MNm^{-2}$ for two possible types of weld defects.

(a)



$a = 5\ mm$
$t = 10\ mm$

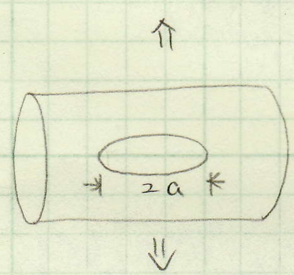
$Y = 3$ from Fig. B.3

$K = Y\sigma\sqrt{\pi a}$

$K = (3)(100\times10^6\ N/m^2)(\pi\times 0.005\ m)^{1/2}$

$K = \underline{37.6\ MNm^{-3/2}}$ +5

(b)



$a = 20\ mm$

$2a = 2\times 20\ mm = 40\ mm$

$K = \sigma\sqrt{\pi a}$

$K = (100\times10^6\ N/m^2)(\pi\times 0.020\ m)^{1/2}$

$K = 25.1\ MN\ m^{-3/2}$

+5

part (a) is more dangerous since fracture factor K is greater

16.1 Why are ceramics usually much stronger in compression than in tension? Al_2O_3 has a fracture toughness K_c of ~ $3\ MN\ m^{-3/2}$. A batch of Al_2O_3 samples is found to contain surface flaws about 30 μm deep. Estimate

(a) tensile stength

$$K_c = \sigma_{TS} \sqrt{\pi a_c}$$

$$\sigma_{TS} = \frac{K_c}{\sqrt{\pi a_c}} = \frac{(3 \times 10^6)}{\sqrt{\pi \times (30 \times 10^{-6})}} = \underline{\underline{309\ MPa}}$$

(b) compressive strength

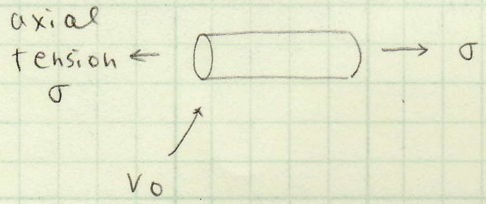
$$\sigma_C = 15\, \sigma_{TS}$$

$$\sigma_C = 15 \times 309\ MPa = \underline{\underline{4635\ MPa}}$$

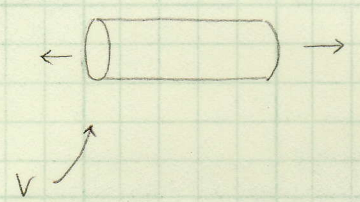
Cracks in compression propagate stably and parallel to compressive forces.

+10

16.3 In order to test the strength of a ceramic, cylindrical specimens of length 25 mm and diameter 5 mm are put into axial tension. The tensile stress σ which causes 50 % of speciments to break is 120 MNm^{-2}. Cylindrical Ceramic components of length 50 mm and diameter 11 mm are required to withstand an axial stress σ′ with a survival propability of 99 %. Given that m = 5, use eqn. 16.7 to determine σ′



l = 25 mm
r = 2.5 mm
P_s = 0.50
σ = 120 MPa



l = 50 mm
r = 5.5 mm
σ = σ′
P_s = 0.99
m = 5

Solve for σ′ ?

$V_0 = \frac{1}{4}\pi(2r)^2 l$

$= \frac{1}{4}\pi(2\times 2.5)^2 \times 25$

$= 491 \text{ mm}^3$

$V = \frac{1}{4}\pi(2r)^2 l$

$= \frac{1}{4}\pi(2\times 5.5)^2 \times 50$

$= 4752 \text{ mm}^3$

m = 5
σ = 120

$PS(V_0) = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] = 0.5 \Rightarrow \sigma_0 = 129 \text{ MN/m}^2$ +5

$PS(V) = \exp\left[\frac{-V}{V_0}\left(\frac{\sigma'}{\sigma_0}\right)^m\right] = 0.99 \Rightarrow \sigma' = 32.6 \text{ MN/m}^2$ +5

16.5

$l_1 = 100\,mm$, $b_1 = d_1 = 10\,mm$
$P_s = 0.5$, $\sigma_r = 300\ MN\,m^{-2}$

$l_2 = 50\,mm$, $b_2 = d_2 = 5\,mm$

$\sigma_{TS} = \sigma_r / 1.73$, $m = 10$

$$P_s(V_0) = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m \right] = 0.5 \Rightarrow \sigma_0 = 180\ MPa$$ +5

$$P_s(V) = \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m \right] = 1 - 10^{-6} \Rightarrow \sigma = 55.7\ MPa$$ +5

$P_s = 1 - P_f$