

and I. Madhusudan, *IEEE Trans. Compon. Packag. Technol.*, **25**, 584, 2002.

Business Week, November 11, 2004.

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conductivity of a sheet of rigid, extruded polystyrene is reported to be $k = 0.029 \text{ W/m} \cdot \text{K}$. The temperature difference across a 20-mm-thick sheet of material is $T_1 - T_2 = 10^\circ\text{C}$.

1.5 Determine the heat flux through a $2 \text{ m} \times 2 \text{ m}$ sheet of material?

1.6 Determine the rate of heat transfer through the sheet of material?

1.7 A wall, which has a surface area of 20 m^2 and is 20 mm thick, separates conditioned room air from outdoor air. The temperature of the inner surface of the wall is maintained at 25°C , and the thermal conductivity of the concrete is $1 \text{ W/m} \cdot \text{K}$.

1.8 Determine the heat loss through the wall for outdoor temperatures ranging from -15°C to 38°C , which correspond to winter and summer extremes, respectively. Display your results graphically.

1.9 Graph, also plot the heat loss as a function of the inner surface temperature for wall materials having thermal conductivities of 0.75 and $1.25 \text{ W/m} \cdot \text{K}$. Display the family of curves you have obtained.

1.10 A concrete slab of a basement is 11 m long, 8 m wide, and 20 cm thick. During the winter, temperatures are typically 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of $1.4 \text{ W/m} \cdot \text{K}$, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.01/\text{MJ}$, what is the daily heating cost?

1.11 Determine the heat loss through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C , respectively. It has been determined to be 40 W/m^2 . What is the thermal conductivity of the wood?

1.12 The inner and outer surface temperatures of a glass window 5 mm thick are 15 and 5°C . What is the heat

loss through a window that is 1 m by 3 m on a side? The thermal conductivity of glass is $1.4 \text{ W/m} \cdot \text{K}$.

1.6 A glass window of width $W = 1 \text{ m}$ and height $H = 2 \text{ m}$ is 5 mm thick and has a thermal conductivity of $k_g = 1.4 \text{ W/m} \cdot \text{K}$. If the inner and outer surface temperatures of the glass are 15°C and -20°C , respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of 10°C and -15°C , what is the rate of heat loss from a $1 \text{ m} \times 2 \text{ m}$ window? The thermal conductivity of air is $k_a = 0.024 \text{ W/m} \cdot \text{K}$.

1.7 A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation ($k = 0.030 \text{ W/m} \cdot \text{K}$) that must be applied to the top and side walls to ensure a heat load of less than 500 W , when the inner and outer surfaces are -10 and 35°C ?

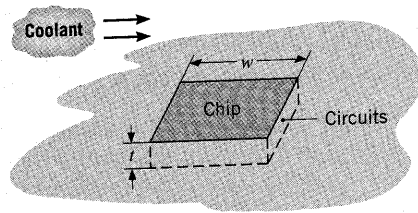
1.8 An inexpensive food and beverage container is fabricated from 25-mm -thick polystyrene ($k = 0.023 \text{ W/m} \cdot \text{K}$) and has interior dimensions of $0.8 \text{ m} \times 0.6 \text{ m} \times 0.6 \text{ m}$. Under conditions for which an inner surface temperature of approximately 2°C is maintained by an ice-water mixture and an outer surface temperature of 20°C is maintained by the ambient, what is the heat flux through the container wall? Assuming negligible heat gain through the $0.8 \text{ m} \times 0.6 \text{ m}$ base of the cooler, what is the total heat load for the prescribed conditions?

1.9 What is the thickness required of a masonry wall having thermal conductivity $0.75 \text{ W/m} \cdot \text{K}$ if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of $0.25 \text{ W/m} \cdot \text{K}$ and a thickness of 100 mm ? Both walls are subjected to the same surface temperature difference.

1.10 The 5-mm -thick bottom of a 200-mm -diameter pan may be made from aluminum ($k = 240 \text{ W/m} \cdot \text{K}$) or

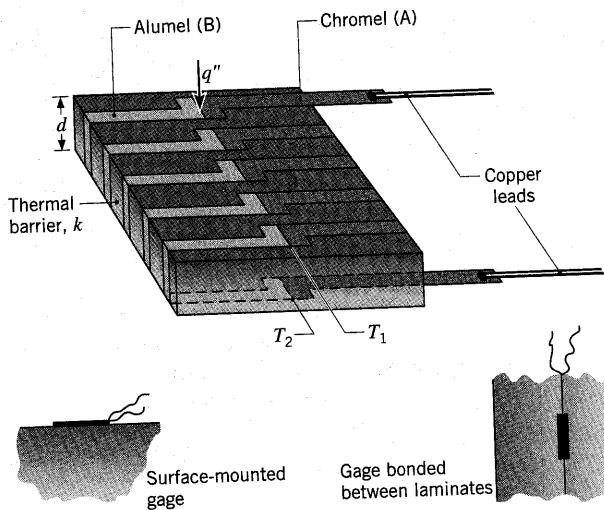
copper ($k = 390 \text{ W/m} \cdot \text{K}$). When used to boil water, the surface of the bottom exposed to the water is nominally at 110°C . If heat is transferred from the stove to the pan at a rate of 600 W , what is the temperature of the surface in contact with the stove for each of the two materials?

- 1.11 A square silicon chip ($k = 150 \text{ W/m} \cdot \text{K}$) is of width $w = 5 \text{ mm}$ on a side and of thickness $t = 1 \text{ mm}$. The chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant.



If 4 W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

- 1.12 A gage for measuring heat flux to a surface or through a laminated material employs five thin-film, chromel/alumel (type K) thermocouples deposited on the upper and lower surfaces of a wafer with a thermal conductivity of $1.4 \text{ W/m} \cdot \text{K}$ and a thickness of 0.25 mm .
- Determine the heat flux q'' through the gage when the voltage output at the copper leads is $350 \mu\text{V}$. The Seebeck coefficient of the type-K thermocouple materials is approximately $40 \mu\text{V}/^\circ\text{C}$.
 - What precaution should you take in using a gage of this nature to measure heat flow through the laminated structure shown?



Convection

- 1.13 You've experienced convection cooling if you've extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C , determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at -5°C with a convection coefficient of $40 \text{ W/m}^2 \cdot \text{K}$ and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of $900 \text{ W/m}^2 \cdot \text{K}$. Which condition would feel colder? Contrast these results with a heat loss of approximately 30 W/m^2 under normal room conditions.

- 1.14 Air at 40°C flows over a long, 25-mm -diameter cylinder with an embedded electrical heater. In a series of test measurements were made of the power per unit length P' , required to maintain the cylinder surface temperature at 300°C for different freestream velocities V of the air. The results are as follows:

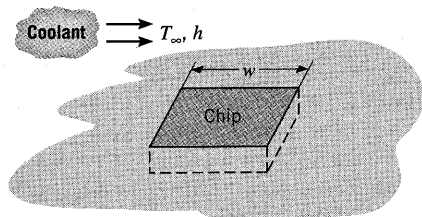
Air velocity, V (m/s)	1	2	4	8	15
Power, P' (W/m)	450	658	983	1507	1900

- Determine the convection coefficient for each velocity, and display your results graphically.
 - Assuming the dependence of the convection coefficient on the velocity to be of the form $h = CV^n$, determine the parameters C and n from the results of part (a).
- 1.15 An electric resistance heater is embedded in a long cylinder of diameter 30 mm . When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m . When air, also at 25°C , but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m . Calculate and compare the convection coefficients for the flows of water and air.

- 1.16 A cartridge electrical heater is shaped as a cylinder of length $L = 200 \text{ mm}$ and outer diameter $D = 20 \text{ mm}$. Under normal operating conditions the heater dissipates 2 kW while submerged in a water flow that is at 20°C and provides a convection heat transfer coefficient $h = 5000 \text{ W/m}^2 \cdot \text{K}$. Neglecting heat transfer from the ends of the heater, determine its surface temperature. If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air that is also at 20°C but for which $h = 10 \text{ W/m}^2 \cdot \text{K}$. What is the corresponding surface temperature? What are the consequences of such an event?

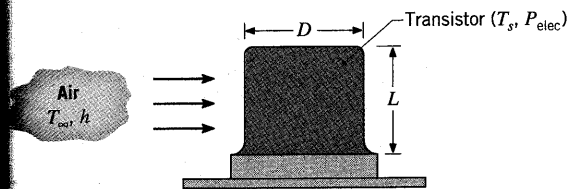
1.17 A common procedure for measuring the velocity of an air stream involves insertion of an electrically heated wire (called a *hot-wire anemometer*) into the air flow, with the axis of the wire oriented perpendicular to the flow direction. The electrical energy dissipated in the wire is assumed to be transferred to the air by forced convection. Hence, for a prescribed electrical power, the temperature of the wire depends on the convection coefficient, which, in turn, depends on the velocity of the air. Consider a wire of length $L = 20$ mm and diameter $D = 0.5$ mm, for which a calibration of the form, $V = 6.25 \times 10^{-5} h^2$, has been determined. The velocity V and the convection coefficient h have units of m/s and $\text{W/m}^2 \cdot \text{K}$, respectively. In an application involving air at a temperature of $T_\infty = 25^\circ\text{C}$, the surface temperature of the anemometer is maintained at $T_s = 75^\circ\text{C}$ with a voltage drop of 5 V and an electric current of 0.1 A. What is the velocity of the air?

1.18 A square isothermal chip is of width $w = 5$ mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at $T_\infty = 15^\circ\text{C}$. From reliability considerations, the chip temperature must not exceed $T = 85^\circ\text{C}$.



If the coolant is air and the corresponding convection coefficient is $h = 200 \text{ W/m}^2 \cdot \text{K}$, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which $h = 3000 \text{ W/m}^2 \cdot \text{K}$, what is the maximum allowable power?

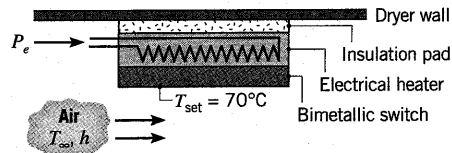
1.19 The case of a power transistor, which is of length $L = 10$ mm and diameter $D = 12$ mm, is cooled by an air stream of temperature $T_\infty = 25^\circ\text{C}$.



Under conditions for which the air maintains an average convection coefficient of $h = 100 \text{ W/m}^2 \cdot \text{K}$ on the surface of the case, what is the maximum allowable power dissipation if the surface temperature is not to exceed 85°C ?

1.20 The use of impinging air jets is proposed as a means of effectively cooling high-power logic chips in a computer. However, before the technique can be implemented, the convection coefficient associated with jet impingement on a chip surface must be known. Design an experiment that could be used to determine convection coefficients associated with air jet impingement on a chip measuring approximately 10 mm by 10 mm on a side.

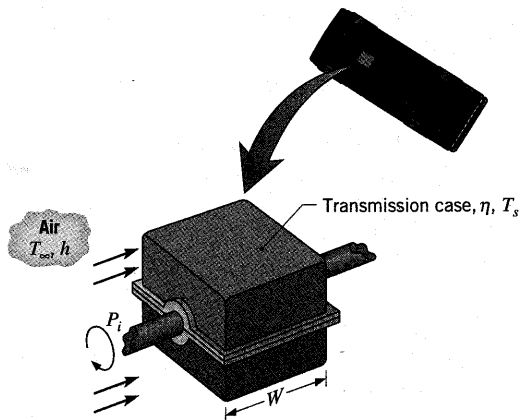
1.21 The temperature controller for a clothes dryer consists of a bimetallic switch mounted on an electrical heater attached to a wall-mounted insulation pad.



The switch is set to open at 70°C , the maximum dryer air temperature. In order to operate the dryer at a lower air temperature, sufficient power is supplied to the heater such that the switch reaches 70°C (T_{set}) when the air temperature T is less than T_{set} . If the convection heat transfer coefficient between the air and the exposed switch surface of 30 mm^2 is $25 \text{ W/m}^2 \cdot \text{K}$, how much heater power P_e is required when the desired dryer air temperature is $T_\infty = 50^\circ\text{C}$?

1.22 The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is 225°C and the change in plate temperature with time (dT/dt) is -0.022 K/s . The ambient air temperature is 25°C and the plate measures $0.3 \times 0.3 \text{ m}$ with a mass of 3.75 kg and a specific heat of $2770 \text{ J/kg} \cdot \text{K}$.

1.23 A transmission case measures $W = 0.30 \text{ m}$ on a side and receives a power input of $P_i = 150 \text{ hp}$ from the engine.



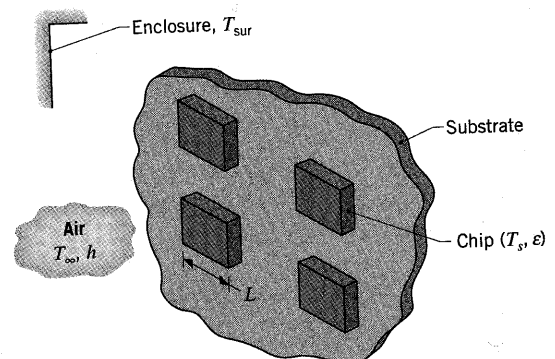
If the transmission efficiency is $\eta = 0.93$ and air flow over the case corresponds to $T_\infty = 30^\circ\text{C}$ and $h = 200 \text{ W/m}^2 \cdot \text{K}$, what is the surface temperature of the transmission?

1.29 If $T_s \approx T_{\text{sur}}$ in Equation 1.9, the radiation heat transfer coefficient may be approximated as

$$h_{r,a} = 4\epsilon\sigma\bar{T}^3$$

where $\bar{T} \equiv (T_s + T_{\text{sur}})/2$. We wish to assess the validity of this approximation by comparing values of h_r and $h_{r,a}$ for the following conditions. In each case represent your results graphically and comment on the validity of the approximation.

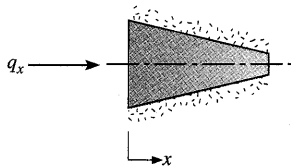
- 1.24** Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at 20°C throughout the year, while the walls of the room are nominally at 27°C and 14°C in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32°C throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately $2 \text{ W/m}^2 \cdot \text{K}$.
- 1.25** A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?
- 1.26** An instrumentation package has a spherical outer surface of diameter $D = 100 \text{ mm}$ and emissivity $\epsilon = 0.25$. The package is placed in a large space simulation chamber whose walls are maintained at 77 K. If operation of the electronic components is restricted to the temperature range $40 \leq T \leq 85^\circ\text{C}$, what is the range of acceptable power dissipation for the package? Display your results graphically, showing also the effect of variations in the emissivity by considering values of 0.20 and 0.30.
- 1.27** Consider the conditions of Problem 1.22. However, now the plate is in a vacuum with a surrounding temperature of 25°C . What is the emissivity of the plate? What is the rate at which radiation is emitted by the surface?
- 1.28** An overhead 25-m-long, uninsulated industrial steam pipe of 100 mm diameter is routed through a building whose walls and air are at 25°C . Pressurized steam maintains a pipe surface temperature of 150°C , and the coefficient associated with natural convection is $h = 10 \text{ W/m}^2 \cdot \text{K}$. The surface emissivity is $\epsilon = 0.8$.
- What is the rate of heat loss from the steam line?
 - If the steam is generated in a gas-fired boiler operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.01$ per MJ, what is the annual cost of heat loss from the line?
- 1.30** Consider the conditions of Problem 1.18. With heat transfer by convection to air, the maximum allowable chip power is found to be 0.35 W. If consideration is also given to net heat transfer by radiation from the chip surface to large surroundings at 15°C , what is the percentage increase in the maximum allowable chip power afforded by this consideration? The chip surface has an emissivity of 0.9.
- 1.31** Chips of width $L = 15 \text{ mm}$ on a side are mounted to a substrate that is installed in an enclosure whose walls and air are maintained at a temperature of $T_{\text{sur}} = T_\infty = 25^\circ\text{C}$. The chips have an emissivity of $\epsilon = 0.60$ and a maximum allowable temperature of $T_s = 85^\circ\text{C}$.



- (a) If heat is rejected from the chips by radiation and natural convection, what is the maximum operating

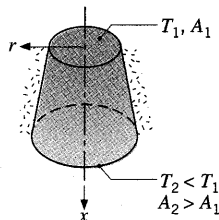
2.3 A spherical shell with inner radius r_1 and outer radius r_2 has surface temperatures T_1 and T_2 , respectively, where $T_1 > T_2$. Sketch the temperature distribution on T - r coordinates assuming steady-state, one-dimensional conduction with constant properties. Briefly justify the shape of your curve.

2.4 Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.



Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in kelvins, and x in meters.

2.5 A solid, truncated cone serves as a support for a system that maintains the top (truncated) face of the cone at a temperature T_1 , while the base of the cone is at a temperature $T_2 < T_1$.



The thermal conductivity of the solid depends on temperature according to the relation $k = k_o - aT$, where a is a positive constant, and the sides of the cone are well insulated. Do the following quantities increase, decrease, or remain the same with increasing x : the heat transfer rate q_x , the heat flux q_x'' , the thermal conductivity k , and the temperature gradient dT/dx ?

2.6 To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as

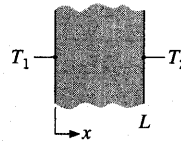
$$k = k_o + aT$$

where k_o is a positive constant and a is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in

a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

2.7 A young engineer is asked to design a thermal protection barrier for a sensitive electronic device that might be exposed to irradiation from a high-powered infrared laser. Having learned as a student that a low thermal conductivity material provides good insulating characteristics, the engineer specifies use of a nanostructured aerogel, characterized by a thermal conductivity of $k_a = 0.005$ W/m · K, for the protective barrier. The engineer's boss questions the wisdom of selecting the aerogel because it has a low thermal conductivity. Consider the sudden laser irradiation of (a) pure aluminum, (b) glass, and (c) aerogel. The laser provides irradiation of $G = 10 \times 10^6$ W/m². The absorptivities of the materials are $\alpha = 0.2, 0.9,$ and 0.8 for the aluminum, glass, and aerogel, respectively, and the initial temperature of the barrier is $T_i = 300$ K. Explain why the boss is concerned. *Hint:* All materials experience thermal expansion (or contraction), and local stresses that develop within a material are, to a first approximation, proportional to the local temperature gradient.

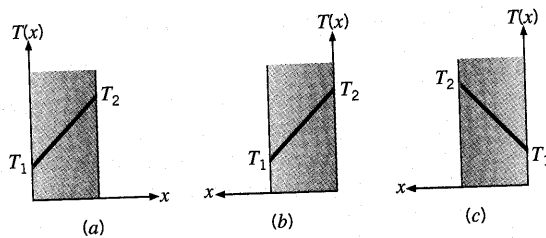
2.8 Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.



Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

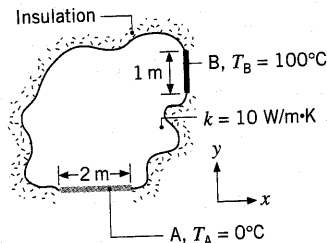
Case	T_1 (°C)	T_2 (°C)	dT/dx (K/m)
1	50	-20	
2	-30	-10	
3	70		160
4		40	-80
5		30	200

2.9 Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux q_x'' and the temperature gradient dT/dx for the coordinate systems shown.



2.10 A cylinder of radius r_o , length L , and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞ . At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

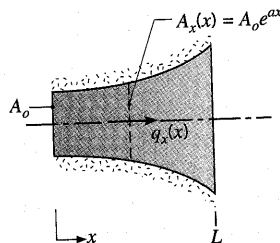


2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m·K) that are 1 m long and have a cross-sectional area of 0.005 m². Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft-pipeline joint ($x = 0$) and at the shaft-ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



- (a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.
- (b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where \dot{q}_o is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through $L = 10$ -mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The thermal conductivities of the aerogel and polycarbonate are $k_{ag} = 0.014$ W/m·K and $k_{pc} = 0.21$ W/m·K, respectively. Evaluate the thermal conductivity of the soda lime glass at 300 K. If the aircraft has 130 windows and the cost to heat the cabin air is \$1/kW·h, compare the costs associated with the heat loss through the windows for an 8-hour intercontinental flight.

2.15 Gold is commonly used in semiconductor packaging to form interconnections (also known as interconnects) that carry electrical signals between different devices in the package. In addition to being a good electrical conductor, gold interconnects are also effective at protecting the heat-generating devices to which they are attached by conducting thermal energy away from the devices to surrounding, cooler regions. Consider a thin film of gold that has a cross section of 60 nm × 250 nm.

- (a) For an applied temperature difference of 20°C, determine the energy conducted along a 1-μm-long thin-film interconnect. Evaluate properties at 300 K.
- (b) Plot the lengthwise (in the 1-μm direction) and spanwise (in the thinnest direction) thermal conductivities of the gold film as a function of the film thickness, L , for $30 \leq L \leq 140$ nm.

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn't the thickness of the insulating material that counts, it's the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood or just 6 in. of the manufacturer's insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don't know the R-value is defined as L/k where L (in.) is the thickness of the insulation and k (Btu·in./hr·ft²·°F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two

the volumetric rate of heat generation \dot{q} in W/m^3 ?

the surface heat fluxes, $q''(-L)$ and $q''(+L)$. How are these fluxes related to the heat generation rate?

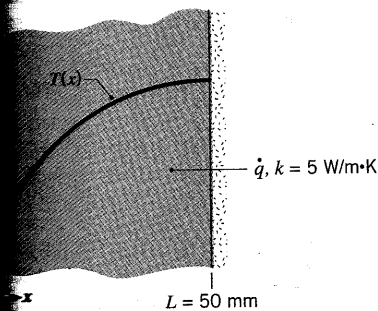
the convection coefficients for the surfaces at $x = -L$ and $x = +L$?

an expression for the heat flux distribution, $q''(x)$. Is the heat flux zero at any location? Explain the significant features of the distribution.

the source of the heat generation is suddenly removed ($\dot{q} = 0$), what is the rate of change of temperature in the wall at this instant?

the temperature will the wall eventually reach? How much energy must be removed by convection per unit area of the wall (J/m^2) to reach this temperature? The density and specific heat of the wall material are $\rho = 2600 \text{ kg/m}^3$ and $c_p = 800 \text{ J/kg} \cdot \text{K}$, respectively.

For steady-state conduction with uniform heat generation occurs in a plane wall with a thickness L and a constant thermal conductivity k . For these conditions, the temperature distribution is of the form, $T(x) = a + bx + cx^2$. The surface temperature of $T(0) \equiv T_o = 120^\circ\text{C}$ and convection with a fluid for which $T_\infty = 20^\circ\text{C}$ and $h = 1000 \text{ W/m}^2 \cdot \text{K}$. The surface at $x = L$ is well insulated.



an overall energy balance to the wall, calculate the internal energy generation rate, \dot{q} .

the coefficients a , b , and c by applying boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

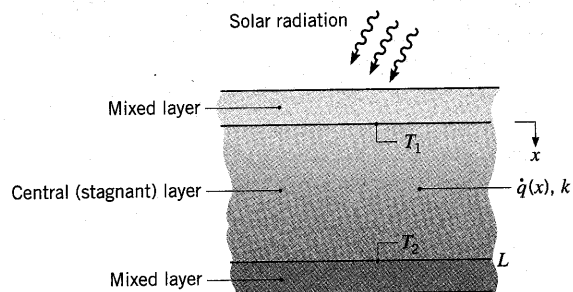
conditions for which the convection coefficient is halved, but the internal energy generation rate is unchanged. Determine the new values of a , b , and c , and use the results to plot the temperature distribution. Hint: recognize that $T(0)$ is not to change.

- (d) Under conditions for which the internal energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500 \text{ W/m}^2 \cdot \text{K}$), determine the new values of a , b , and c and plot the corresponding temperature distribution. Referring to the results of parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and \dot{q} on the distributions.

- 2.27 A salt-gradient solar pond is a shallow body of water that consists of three distinct fluid layers and is used to collect solar energy. The upper- and lower-most layers are well mixed and serve to maintain the upper and lower surfaces of the central layer at uniform temperatures T_1 and T_2 , where $T_2 > T_1$. Although there is bulk fluid motion in the mixed layers, there is no such motion in the central layer. Consider conditions for which solar radiation absorption in the central layer provides nonuniform heat generation of the form $\dot{q} = Ae^{-ax}$, and the temperature distribution in the central layer is

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

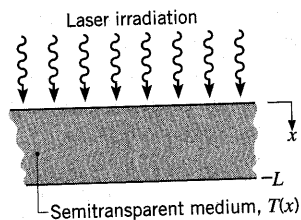
The quantities A (W/m^3), a ($1/\text{m}$), B (K/m), and C (K) are known constants having the prescribed units, and k is the thermal conductivity, which is also constant.



- (a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from the central layer to the upper mixed layer.
- (b) Determine whether conditions are steady or transient.
- (c) Obtain an expression for the rate at which thermal energy is generated in the entire central layer, per unit surface area.
- 2.28 The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

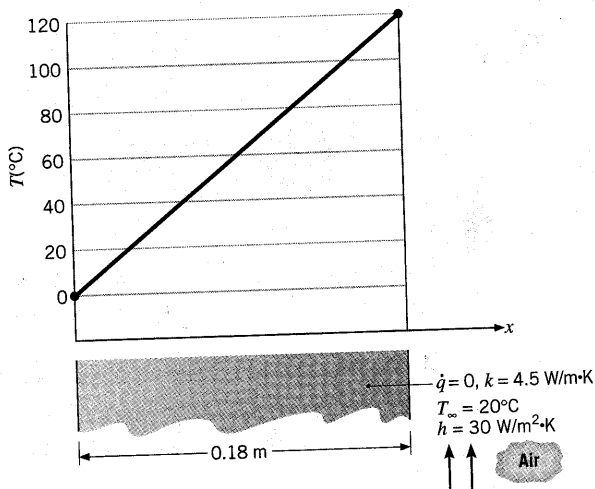
where A , a , B , and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $\dot{q}(x)$.



- Obtain expressions for the conduction heat fluxes at the front and rear surfaces.
- Derive an expression for $\dot{q}(x)$.
- Derive an expression for the rate at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

2.29 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity k and thickness L is of the form $T = ax^3 + bx^2 + cx + d$. Derive expressions for the heat generation rate per unit volume in the wall and the heat fluxes at the two wall faces ($x = 0, L$).

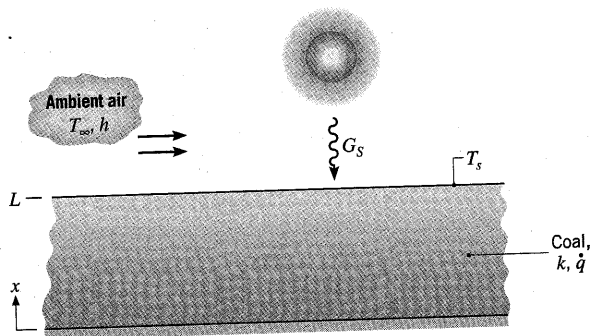
2.30 One-dimensional, steady-state conduction with no internal energy generation is occurring in a plane wall of constant thermal conductivity.



- Is the prescribed temperature distribution possible? Briefly explain your reasoning.
- With the temperature at $x = 0$ and the fluid temperature fixed at $T(0) = 0^\circ\text{C}$ and $T_\infty = 20^\circ\text{C}$,

respectively, compute and plot the temperature at $x = L, T(L)$, as a function of h for $10 \leq h \leq 100 \text{ W/m}^2 \cdot \text{K}$. Briefly explain your results.

2.31 A plane layer of coal of thickness $L = 1 \text{ m}$ experiences uniform volumetric generation at a rate of $\dot{q} = 20 \text{ W/m}^3$ due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which $h = 5 \text{ W/m}^2 \cdot \text{K}$ and $T_\infty = 25^\circ\text{C}$, while receiving solar irradiation in the amount $G_s = 400 \text{ W/m}^2$. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each $\alpha_s = \varepsilon = 0.95$.



- Write the steady-state form of the heat diffusion equation for the layer of coal. Verify that this equation is satisfied by a temperature distribution of the form

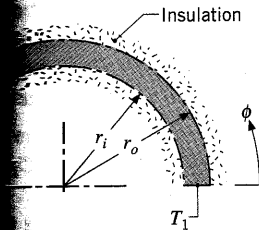
$$T(x) = T_s + \frac{\dot{q}L^2}{2k} \left(1 - \frac{x^2}{L^2} \right)$$

From this distribution, what can you say about conditions at the bottom surface ($x = 0$)? Sketch the temperature distribution and label key features.

- Obtain an expression for the rate of heat transfer by conduction per unit area at $x = L$. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for T_s . Evaluate T_s and $T(0)$ for the prescribed conditions.

- Daily average values of G_s and h depend on a number of factors such as time of year, cloud cover, and wind conditions. For $h = 5 \text{ W/m}^2 \cdot \text{K}$, compute and plot T_s and $T(0)$ as a function of G_s for $50 \leq G_s \leq 500 \text{ W/m}^2$. For $G_s = 400 \text{ W/m}^2$, compute and plot T_s and $T(0)$ as a function of h for $5 \leq h \leq 50 \text{ W/m}^2 \cdot \text{K}$.

2.32 The cylindrical system illustrated has negligible variation of temperature in the r and z directions. Assume that $\Delta r = r_o - r_i$ is small compared to r_i and denote the length in the z direction, normal to the page, as L .



With a properly defined control volume showing energy generation and storage effects, derive the differential equation that prescribes the temperature with the angular coordinate ϕ . Compare your result with Equation 2.24.

Under conditions with no internal heat generation and constant properties, determine the temperature distribution $T(\phi)$ in terms of the constants r_i and r_o . Is this distribution linear in ϕ ?

For conditions of part (b) write the expression for the heat rate q_ϕ .

Using a differential control volume in the form of a spherical shell, derive the heat diffusion equation in three-dimensional, cylindrical, radial coordinates with internal heat generation. Compare your result with Equation 2.24.

Using a differential control volume in the form of a spherical shell, derive the heat diffusion equation in three-dimensional, spherical, radial coordinate coordinates with internal heat generation. Compare your result with Equation 2.27.

Using the heat diffusion equation, Equation 2.24, for one-dimensional radial coordinates beginning with the differential control volume shown in Figure 2.12.

Using the heat diffusion equation, Equation 2.27, for one-dimensional radial coordinates beginning with the differential control volume shown in Figure 2.13.

A cylindrical rod is wrapped with insulation of inner and outer radii r_i and r_o , respectively. At a particular instant the temperature distribution in the insulation is known to be

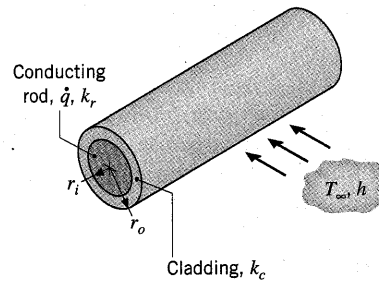
$$T(r) = C_1 \ln\left(\frac{r}{r_o}\right) + C_2$$

Is this steady-state or transient? How do the heat rates vary with radius?

A cylindrical tube of inner and outer radii r_1 and r_2 , respectively, is exposed to uniform temperatures T_1 and T_2 at the inner and outer surfaces, while thermal energy generation is occurring within the tube wall. Consider steady-state conditions for the tube. Is it possible to maintain a linear radial

temperature distribution in the wall? If so, what special conditions must exist?

- 2.39 Passage of an electric current through a long conducting rod of radius r_i and thermal conductivity k_r results in uniform volumetric heating at a rate of \dot{q} . The conducting rod is wrapped in an electrically nonconducting cladding material of outer radius r_o and thermal conductivity k_c , and convection cooling is provided by an adjoining fluid.



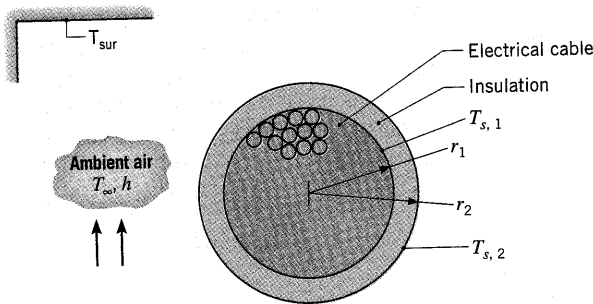
For steady-state conditions, write appropriate forms of the heat equations for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

- 2.40 Two-dimensional, steady-state conduction occurs in a hollow cylindrical solid of thermal conductivity $k = 16 \text{ W/m} \cdot \text{K}$, outer radius $r_o = 1 \text{ m}$, and overall length $2z_o = 5 \text{ m}$, where the origin of the coordinate system is located at the midpoint of the centerline. The inner surface of the cylinder is insulated, and the temperature distribution within the cylinder has the form $T(r, z) = a + br^2 + c \ln r + dz^2$, where $a = 20^\circ\text{C}$, $b = 150^\circ\text{C/m}^2$, $c = -12^\circ\text{C}$, $d = -300^\circ\text{C/m}^2$ and r and z are in meters.

- Determine the inner radius r_i of the cylinder.
- Obtain an expression for the volumetric rate of heat generation, $\dot{q}(\text{W/m}^3)$.
- Determine the axial distribution of the heat flux at the outer surface, $q_r''(r_o, z)$. What is the heat rate at the outer surface? Is it into or out of the cylinder?
- Determine the radial distribution of the heat flux at the end faces of the cylinder, $q_z''(r, z_o)$ and $q_z''(r, -z_o)$. What are the corresponding heat rates? Are they into or out of the cylinder?
- Verify that your results are consistent with an overall energy balance on the cylinder.

- 2.41 An electric cable of radius r_1 and thermal conductivity k_c is enclosed by an insulating sleeve whose outer surface is of radius r_2 and experiences convection heat transfer and radiation exchange with the adjoining air and large surroundings, respectively. When electric current passes

through the cable, thermal energy is generated within the cable at a volumetric rate \dot{q} .



- (a) Write the steady-state forms of the heat diffusion equation for the insulation and the cable. Verify that these equations are satisfied by the following temperature distributions:

$$\text{Insulation: } T(r) = T_{s,2} + (T_{s,1} - T_{s,2}) \frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

$$\text{Cable: } T(r) = T_{s,1} + \frac{\dot{q} r_1^2}{4k_c} \left(1 - \frac{r^2}{r_1^2} \right)$$

Sketch the temperature distribution, $T(r)$, in the cable and the sleeve, labeling key features.

- (b) Applying Fourier's law, show that the rate of conduction heat transfer per unit length through the sleeve may be expressed as

$$q'_r = \frac{2\pi k_s (T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Applying an energy balance to a control surface placed around the cable, obtain an alternative expression for q'_r , expressing your result in terms of \dot{q} and r_1 .

- (c) Applying an energy balance to a control surface placed around the outer surface of the sleeve, obtain an expression from which $T_{s,2}$ may be determined as a function of \dot{q} , r_1 , h , T_∞ , ε , and T_{sur} .
- (d) Consider conditions for which 250 A are passing through a cable having an electric resistance per unit length of $R_e' = 0.005 \Omega/\text{m}$, a radius of $r_1 = 15$ mm, and a thermal conductivity of $k_c = 200 \text{ W/m} \cdot \text{K}$. For $k_s = 0.15 \text{ W/m} \cdot \text{K}$, $r_2 = 15.5$ mm, $h = 25 \text{ W/m}^2 \cdot \text{K}$, $\varepsilon = 0.9$, $T_\infty = 25^\circ\text{C}$, and $T_{\text{sur}} = 35^\circ\text{C}$, evaluate the surface temperatures, $T_{s,1}$ and $T_{s,2}$, as well as the temperature T_o at the centerline of the cable.

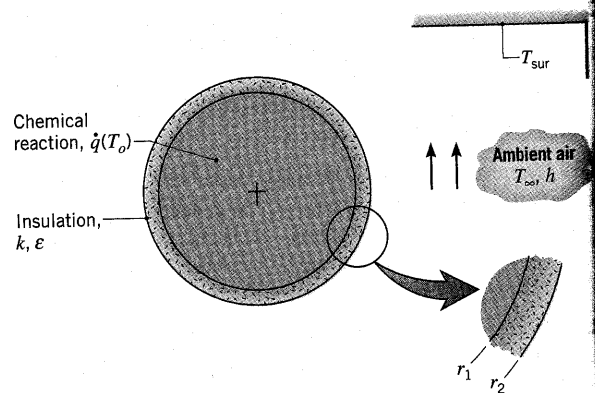
- (e) With all other conditions remaining the same, compute and plot T_o , $T_{s,1}$, and $T_{s,2}$ as a function of r_2 for $15.5 \leq r_2 \leq 20$ mm.

- 2.42 A spherical shell of inner and outer radii r_i and r_o , respectively, contains heat-dissipating components, and at a particular instant the temperature distribution in the shell is known to be of the form

$$T(r) = \frac{C_1}{r} + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

- 2.43 A chemically reacting mixture is stored in a thin-walled spherical container of radius $r_1 = 200$ mm, and the exothermic reaction generates heat at a uniform, but temperature-dependent volumetric rate of $\dot{q} = \dot{q}_o \exp(-A/T_o)$, where $\dot{q}_o = 5000 \text{ W/m}^3$, $A = 75 \text{ K}$, and T_o is the mixture temperature in kelvins. The vessel is enclosed by an insulating material of outer radius r_2 , thermal conductivity k , and emissivity ε . The outer surface of the insulation experiences convection heat transfer and net radiation exchange with the adjoining air and large surroundings, respectively.



- (a) Write the steady-state form of the heat diffusion equation for the insulation. Verify that this equation is satisfied by the temperature distribution

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2}) \left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$$

Sketch the temperature distribution, $T(r)$, labeling key features.

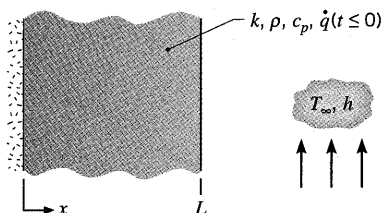
- (b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as

$$q_r = \frac{4\pi k (T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r , expressing your result in terms of \dot{q} and r_1 .

- (a) By applying the conservation of energy principle to spherical control volume A, which is placed at an arbitrary location within the sphere, determine a relationship between the temperature gradient, dT/dr , and the local radius, r , for $0 \leq r \leq r_1$.
- (b) By applying the conservation of energy principle to spherical control volume B, which is placed at an arbitrary location within the spherical shell, determine a relationship between the temperature gradient, dT/dr , and the local radius, r , for $r_1 \leq r \leq r_2$.
- (c) On $T - r$ coordinates, sketch the temperature distribution over the range $0 \leq r \leq r_2$.

A plane wall of thickness $L = 0.1$ m experiences uniform volumetric heating at a rate \dot{q} . One surface of the wall ($x = 0$) is insulated, while the other surface is exposed to a fluid at $T_\infty = 20^\circ\text{C}$, with convection heat transfer characterized by $h = 1000$ W/m² · K. Initially, the temperature distribution in the wall is $T(x, 0) = a + bx^2$, where $a = 300^\circ\text{C}$, $b = -1.0 \times 10^4$ C/m² and x is in meters. Suddenly, the volumetric heat generation is deactivated ($\dot{q} = 0$ for $t \geq 0$), while convection heat transfer continues to occur at $x = L$. The properties of the wall are $\rho = 7000$ kg/m³, $c_p = 450$ J/kg · K, and $k = 90$ W/m · K.



- (a) Determine the magnitude of the volumetric energy generation rate \dot{q} associated with the initial condition ($t < 0$).
- (b) On $T - x$ coordinates, sketch the temperature distribution for the following conditions: initial condition ($t < 0$), steady-state condition ($t \rightarrow \infty$), and two intermediate conditions.
- (c) On $q_x'' - t$ coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q_x''(L, t)$. Calculate the corresponding value of the heat flux at $t = 0$, $q_x''(L, 0)$.
- (d) Calculate the amount of energy removed from the wall per unit area (J/m²) by the fluid stream as the wall cools from its initial to steady-state condition.

A plane wall that is insulated on one side ($x = 0$) is initially at a uniform temperature T_i , when its exposed surface at $x = L$ is suddenly raised to a temperature T_s .

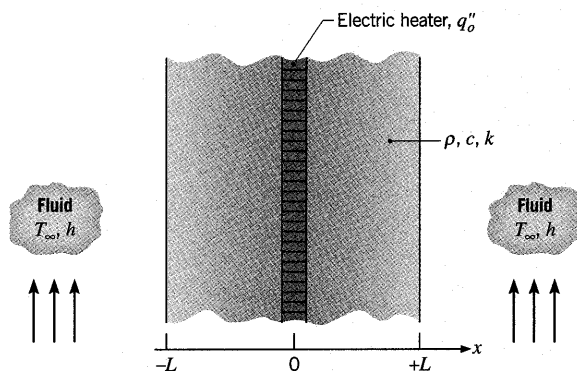
- (a) Verify that the following equation satisfies the heat equation and boundary conditions:

$$\frac{T(x, t) - T_s}{T_i - T_s} = C_1 \exp\left(-\frac{\pi^2 \alpha t}{4 L^2}\right) \cos\left(\frac{\pi x}{2 L}\right)$$

where C_1 is a constant and α is the thermal diffusivity.

- (b) Obtain expressions for the heat flux at $x = 0$ and $x = L$.
- (c) Sketch the temperature distribution $T(x)$ at $t = 0$, at $t \rightarrow \infty$, and at an intermediate time. Sketch the variation with time of the heat flux at $x = L$, $q_x''(t)$.
- (d) What effect does α have on the thermal response of the material to a change in surface temperature?

2.53 A thin electrical heater dissipating 4000 W/m² is sandwiched between two 25-mm-thick plates whose exposed surfaces experience convection with a fluid for which $T_\infty = 20^\circ\text{C}$ and $h = 400$ W/m² · K. The thermophysical properties of the plate material are $\rho = 2500$ kg/m³, $c = 700$ J/kg · K, and $k = 5$ W/m · K.



- (a) On $T - x$ coordinates, sketch the steady-state temperature distribution for $-L \leq x \leq +L$. Calculate values of the temperatures at the surfaces, $x = \pm L$, and the midpoint, $x = 0$. Label this distribution as Case 1, and explain its salient features.
- (b) Consider conditions for which there is a loss of coolant and existence of a nearly adiabatic condition on the $x = +L$ surface. On the $T - x$ coordinates used for part (a), sketch the corresponding steady-state temperature distribution and indicate the temperatures at $x = 0, \pm L$. Label the distribution as Case 2, and explain its key features.
- (c) With the system operating as described in part (b), the surface $x = -L$ also experiences a sudden loss of coolant. This dangerous situation goes undetected for 15 minutes, at which time the power to the heater is deactivated. Assuming no heat losses from the surfaces of the plates, what is the eventual ($t \rightarrow \infty$), uni-