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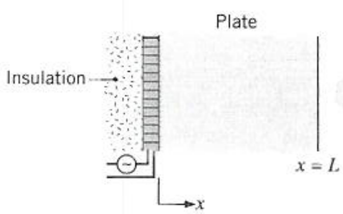
Problems

Qualitative Considerations

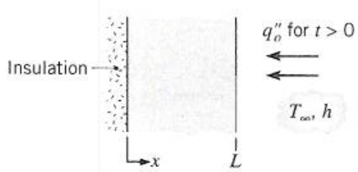
5.1 Consider a thin electrical heater attached to a plate and backed by insulation. Initially, the heater and plate are at the temperature of the ambient air, T_∞. Suddenly, the power to the heater is activated, yielding a constant heat flux q''_o (W/m^2) at the inner surface of the plate.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q''_x(L, t)$ as a function of time.

5.2 The inner surface of a plane wall is insulated while the outer surface is exposed to an airstream at T_∞. The wall is at a uniform temperature corresponding to that of the airstream. Suddenly, a radiation heat source is switched on applying a uniform flux q''_o to the outer surface.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q''_x(L, t)$ as a function of time.

5.3 A microwave oven operates on the principle that application of a high-frequency field causes electrically polarized molecules in food to oscillate. The net effect is a nearly *uniform generation* of thermal energy within the food. Consider the process of cooking a slab of beef of thickness $2L$ in a microwave oven and compare it with cooking in a conventional oven, where *each side* of the slab is *heated by radiation*. In each case the meat is to be heated from 0°C to a *minimum* temperature of 90°C. Base your comparison on a sketch of the temperature distribution at selected times for each of the cooking processes. In particular, consider the time t_0 at which heating is initiated, a time t_1 during the heating process, the time t_2 corresponding to the conclusion of heating, and a time t_3 well into the subsequent cooling process.

5.4 A plate of thickness $2L$, surface area A_s, mass M, and specific heat c_p, initially at a uniform temperature T_i, is suddenly heated on both surfaces by a convection process (T_∞, h) for a period of time t_o, following which the plate is insulated. Assume that the midplane temperature does not reach T_∞ within this period of time.

(a) Assuming $Bi \gg 1$ for the heating process, sketch and label, on T–x coordinates, the following temperature distributions: initial, steady-state ($t \to \infty$), $T(x, t_o)$, and at two intermediate times between $t = t_o$ and $t \to \infty$.

(b) Sketch and label, on T–t coordinates, the midplane and exposed surface temperature distributions.

(c) Repeat parts (a) and (b) assuming $Bi \ll 1$ for the plate.

(d) Derive an expression for the steady-state temperature $T(x, \infty) = T_f$, leaving your result in terms of plate parameters (M, c_p), thermal conditions (T_i, T_∞, h), the surface temperature $T(L, t)$, and the heating time t_o.

Lumped Capacitance Method

5.5 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/m² · K. Assuming the properties of the steel to be $k = 40$ W/m · K, $\rho = 7800$ kg/m³, and $c = 600$ J/kg · K, estimate the time required for the cooling process.

5.6 Consider the steel balls of Problem 5.5, except now the air temperature increases with time as $T_\infty(t) = 325 \text{ K} + at$ where $a = 0.1875$ K/s.

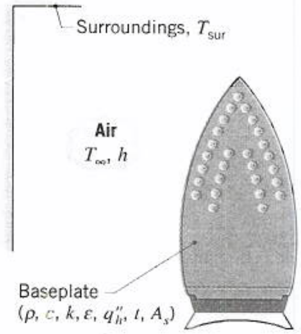
(a) Sketch the ball temperature versus time for $0 \leq t \leq 1$ h. Also show the ambient temperature, T_∞, in your graph. Explain special features of the ball temperature behavior.

(b) Find an expression for the ball temperature as a function of time, $T(t)$, and plot the ball temperature for $0 \leq t \leq 1$ h. Was your sketch correct?

5.7 The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

5.8 A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m · K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_\infty = 100$°C and $h = 3300$ W/m² · K. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance ($\rho c V$) is small compared to that of the steel sphere.

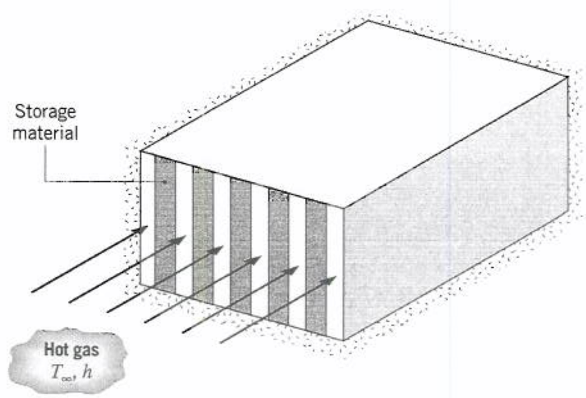
5.9 The base plate of an iron has a thickness of $L = 7$ mm and is made from an aluminum alloy ($\rho = 2800$ kg/m³, $c = 900$ J/kg · K, $k = 180$ W/m · K, $\varepsilon = 0.80$). An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The areas of both the inner and outer surfaces are $A_s = 0.040$ m².



If an approximately uniform heat flux of $q''_h = 1.25 \times 10^4$ W/m² is applied to the inner surface of the base plate and the convection coefficient at the outer surface is $h = 10$ W/m² · K, estimate the time required for the plate to reach a temperature of 135°C. *Hint:* Numerical integration is suggested in order to solve the problem.

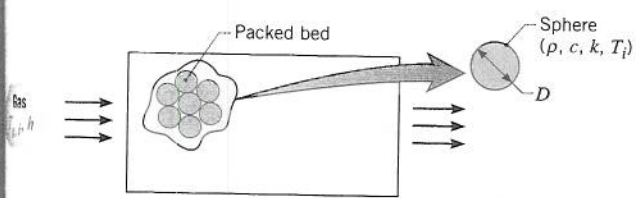
5.10 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 W/m² · K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

5.11 A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.



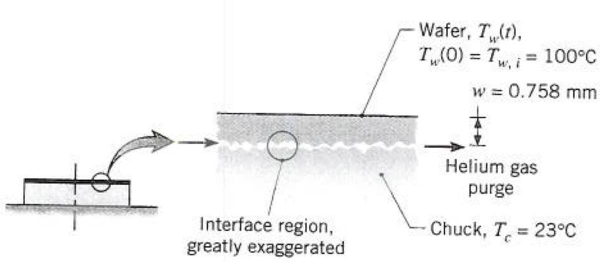
Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_\infty = 600$°C and $h = 100$ W/m² · K throughout the channel. How long will it take to achieve 75% of the maximum possible energy storage? What is the temperature of the aluminum at this time?

5.12 Thermal energy storage systems commonly involve a *packed bed* of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.



Consider a packed bed of 75-mm-diameter aluminum spheres ($\rho = 2700$ kg/m^3, $c = 950$ J/kg · K, $k = 240$ W/m · K) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300$°C. If the initial temperature of the spheres is $T_i = 25$°C and the convection coefficient is $h = 75$ W/m^2 · K, how long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

5.13 A tool used for fabricating semiconductor devices consists of a chuck (thick metallic, cylindrical disk) onto which a very thin silicon wafer ($\rho = 2700$ kg/m^3, $c = 875$ J/kg · K, $k = 177$ W/m · K) is placed by a robotic arm. Once in position, an electric field in the chuck is energized, creating an electrostatic force that holds the wafer firmly to the chuck. To ensure a reproducible thermal contact resistance between the chuck and the wafer from cycle-to-cycle, pressurized helium gas is introduced at the center of the chuck and flows (very slowly) radially outward between the asperities of the interface region.



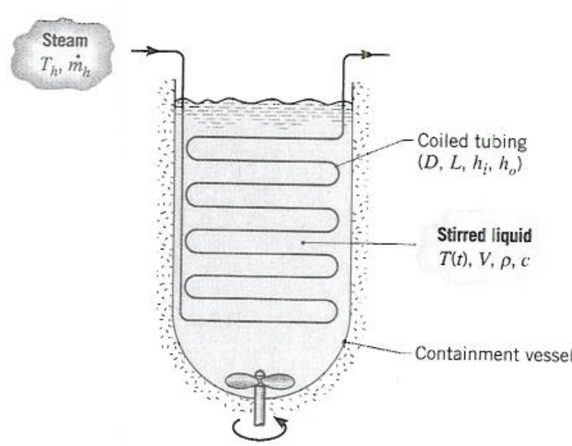
An experiment has been performed under conditions for which the wafer, initially at a uniform temperature $T_{w,i} = 100$°C, is suddenly placed on the chuck, which is at a uniform and constant temperature $T_c = 23$°C. With the wafer in place, the electrostatic force and the helium gas flow are applied. After 15 seconds, the temperature of the wafer is determined to be 33°C. What is the thermal contact resistance $R''_{t,c}$ (m^2 · K/W) between the wafer and chuck? Will the value of $R''_{t,c}$ increase, decrease, or remain the same if air, instead of helium, is used as the purge gas?

5.14 A spherical vessel used as a reactor for producing pharmaceuticals has a 5-mm-thick stainless steel wall ($k = 17$ W/m · K) and an inner diameter of $D_i = 1.0$ m. During production, the vessel is filled with reactants for which $\rho = 1100$ kg/m^3 and $c = 2400$ J/kg · K, while exothermic reactions release energy at a volumetric rate of $\dot{q} = 10^4$ W/m^3. As first approximations, the reactants may be assumed to be well stirred and the thermal capacitance of the vessel may be neglected.

(a) The exterior surface of the vessel is exposed to ambient air ($T_\infty = 25$°C) for which a convection coefficient of $h = 6$ W/m^2 · K may be assumed. If the initial temperature of the reactants is 25°C, what is the temperature of the reactants after five hours of process time? What is the corresponding temperature at the outer surface of the vessel?

(b) Explore the effect of varying the convection coefficient on transient thermal conditions within the reactor.

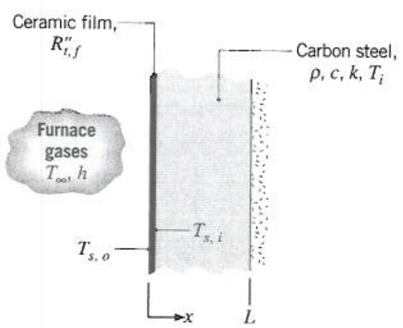
5.15 Batch processes are often used in chemical and pharmaceutical operations to achieve a desired chemical composition for the final product and typically involve a transient heating operation to take the product from room temperature to the desired process temperature.



Consider a situation for which a chemical of density $\rho = 1200\ \text{kg/m}^3$ and specific heat $c = 2200\ \text{J/kg}\cdot\text{K}$ occupies a volume of $V = 2.25\ \text{m}^3$ in an insulated vessel. The chemical is to be heated from room temperature, $T_i = 300$ K, to a process temperature of $T = 450$ K by passing saturated steam at $T_h = 500$ K through a coiled, thin-walled, 20-mm-diameter tube in the vessel. Steam condensation within the tube maintains an interior convection coefficient of $h_i = 10{,}000\ \text{W/m}^2\cdot\text{K}$, while the highly agitated liquid in the stirred vessel maintains an outside convection coefficient of $h_o = 2000\ \text{W/m}^2\cdot\text{K}$.

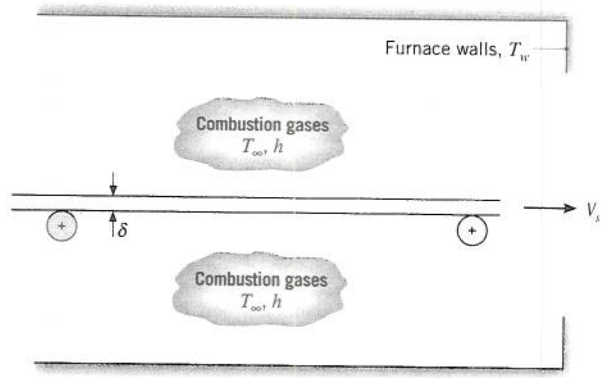
If the chemical is to be heated from 300 to 450 K in 60 minutes, what is the required length L of the submerged tubing?

5.16 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60\ \text{W/m}\cdot\text{K}$, $\rho = 7850\ \text{kg/m}^3$, $c = 430\ \text{J/kg}\cdot\text{K}$) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01\ \text{m}^2\cdot\text{K/W}$. The opposite surface is well insulated from the surroundings.



At furnace start-up the wall is at an initial temperature of $T_i = 300$ K, and combustion gases at $T_\infty = 1300$ K enter the furnace, providing a convection coefficient of $h = 25\ \text{W/m}^2\cdot\text{K}$ at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200$ K? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

5.17 A steel strip of thickness $\delta = 12$ mm is annealed by passing it through a large furnace whose walls are maintained at a temperature T_w corresponding to that of combustion gases flowing through the furnace ($T_w = T_\infty$). The strip, whose density, specific heat, thermal conductivity, and emissivity are $\rho = 7900\ \text{kg/m}^3$, $c_p = 640\ \text{J/kg}\cdot\text{K}$, $k = 30\ \text{W/m}\cdot\text{K}$, and $\varepsilon = 0.7$, respectively, is to be heated from 300°C to 600°C.



(a) For a uniform convection coefficient of $h = 100\ \text{W/m}^2\cdot\text{K}$ and $T_w = T_\infty = 700°\text{C}$, determine the time required to heat the strip. If the strip is moving at 0.5 m/s, how long must the furnace be?

(b) The annealing process may be accelerated (the strip speed increased) by increasing the environmental temperatures. For the furnace length obtained in part (a), determine the strip speed for $T_w = T_\infty = 850°\text{C}$ and $T_w = T_\infty = 1000°\text{C}$. For each set of environmental temperatures (700, 850, and 1000°C), plot the strip temperature as a function of time over the range $25°\text{C} \le T \le 600°\text{C}$. Over this range, also plot the radiation heat transfer coefficient, h_r, as a function of time.

5.18 A long wire of diameter $D = 1$ mm is submerged in an oil bath of temperature $T_\infty = 25°\text{C}$. The wire has an electrical resistance per unit length of $R'_e = 0.01\ \Omega/\text{m}$. If a current of $I = 100$ A flows through the wire and the convection coefficient is $h = 500\ \text{W/m}^2\cdot\text{K}$, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? The properties of the wire are $\rho = 8000\ \text{kg/m}^3$, $c = 500\ \text{J/kg}\cdot\text{K}$, and $k = 20\ \text{W/m}\cdot\text{K}$.

5.19 Consider the system of Problem 5.1 where the temperature of the plate is spacewise isothermal during the transient process.

(a) Obtain an expression for the temperature of the plate as a function of time $T(t)$ in terms of q''_o, T_∞, h, L, and the plate properties ρ and c.

(b) Determine the thermal time constant and the steady-state temperature for a 12-mm-thick plate of pure copper when $T_\infty = 27°\text{C}$, $h = 50\ \text{W/m}^2\cdot\text{K}$, and $q''_o = 5000\ \text{W/m}^2$. Estimate the time required to reach steady-state conditions.

(c) For the conditions of part (b), as well as for $h = 100$ and $200\ \text{W/m}^2\cdot\text{K}$, compute and plot the corresponding temperature histories of the plate for $0 \leq t \leq 2500$ s.

5.20 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

(a) Consider such a system of mass M, specific heat c, and surface area A_s, which is initially in equilibrium with the environment at T_∞. Suddenly, the electronic device is energized such that a constant heat generation $\dot{E}_g$ (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where $\theta \equiv T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \rightarrow \infty$; $\theta_i = T_i - T(\infty)$; T_i = initial temperature of device; R = thermal resistance $1/\bar{h}A_s$; and C = thermal capacitance Mc.

(b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

5.21 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at $T_{sur} = 1000°C$. The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

5.22 A metal sphere of diameter D, which is at a uniform temperature T_i, is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature T_∞ and the surrounding walls at a temperature T_{sur}.

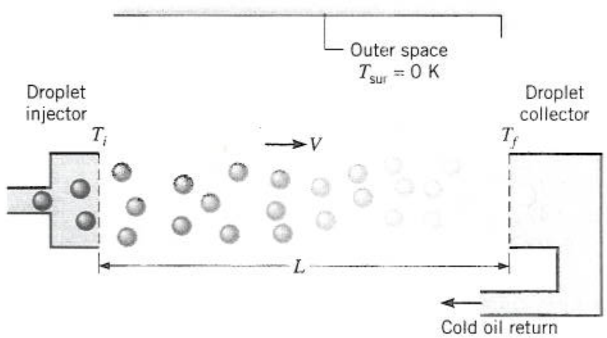
(a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature T.

(b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature T.

(c) How would you go about determining the time required for the sphere to cool to the temperature T if both convection and radiation are of the same order of magnitude?

(d) Consider an anodized aluminum sphere ($\varepsilon = 0.75$) 50 mm in diameter, which is at an initial temperature of $T_i = 800$ K. Both the air and surroundings are at 300 K, and the convection coefficient is 10 W/m$^2 \cdot$K. For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere ($\varepsilon = 0.1$).

5.23 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance L, over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.



Consider conditions for which droplets of emissivity $\varepsilon = 0.95$ and diameter $D = 0.5$ mm are injected at a temperature of $T_i = 500$ K and a velocity of $V = 0.1$ m/s. Properties of the oil are $\rho = 885$ kg/m^3, $c = 1900$ J/kg $\cdot$ K, and $k = 0.145$ W/m $\cdot$ K. Assuming each drop to radiate to deep space at $T_{sur} = 0$ K, determine the distance L required for the droplets to impact the collector at a final temperature of $T_f = 300$ K. What is the amount of thermal energy rejected by each droplet?

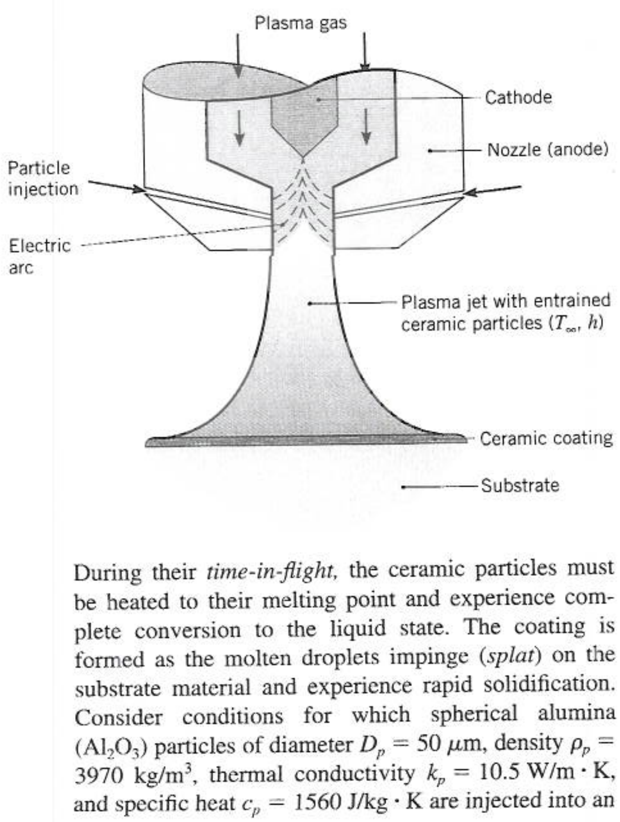
5.24 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with $\rho = 8600$ kg/m^3, $c = 290$ J/kg $\cdot$ K, and $k = 63$ W/m $\cdot$ K.

(a) If cooling is implemented in a large evacuated chamber whose walls are at 25°C, determine the time required to reach the final temperature if the coating is polished and has an emissivity of $\varepsilon = 0.1$. How long would it take if the coating is oxidized and $\varepsilon = 0.6$?

(b) To reduce the time required for cooling, consideration is given to immersion of the sphere in an inert gas stream for which $T_\infty = 25°C$ and $h = 200\ W/m^2 \cdot K$. Neglecting radiation, what is the time required for cooling?

(c) Considering the effect of both radiation and convection, what is the time required for cooling if $h = 200\ W/m^2 \cdot K$ and $\varepsilon = 0.6$? Explore the effect on the cooling time of independently varying h and ε.

5.25 Plasma spray-coating processes are often used to provide surface protection for materials exposed to hostile environments, which induce degradation through factors such as wear, corrosion, or outright thermal failure. *Ceramic* coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle (anode) of a plasma torch, the particles are entrained by the plasma jet, within which they are then accelerated and heated.



During their *time-in-flight*, the ceramic particles must be heated to their melting point and experience complete conversion to the liquid state. The coating is formed as the molten droplets impinge (*splat*) on the substrate material and experience rapid solidification. Consider conditions for which spherical alumina (Al_2O_3) particles of diameter $D_p = 50\ \mu m$, density $\rho_p = 3970\ kg/m^3$, thermal conductivity $k_p = 10.5\ W/m \cdot K$, and specific heat $c_p = 1560\ J/kg \cdot K$ are injected into an arc plasma, which is at $T_\infty = 10{,}000\ K$ and provides a coefficient of $h = 30{,}000\ W/m^2 \cdot K$ for convective heating of the particles. The melting point and latent heat of fusion of alumina are $T_{mp} = 2318\ K$ and $h_{sf} = 3577\ kJ/kg$, respectively.

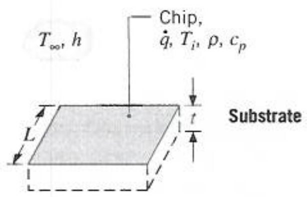
(a) Neglecting radiation, obtain an expression for the time-in-flight, t_{i-f}, required to heat a particle from its initial temperature T_i to its melting point T_{mp}, and, once at the melting point, for the particle to experience complete melting. Evaluate t_{i-f} for $T_i = 300\ K$ and the prescribed heating conditions.

(b) Assuming alumina to have an emissivity of $\varepsilon_p = 0.4$ and the particles to exchange radiation with large surroundings at $T_{sur} = 300\ K$, assess the validity of neglecting radiation.

5.26 Thin film coatings characterized by high resistance to abrasion and fracture may be formed by using microscale composite particles in a plasma spraying process. A spherical particle typically consists of a *ceramic core*, such as tungsten carbide (WC), and a *metallic shell*, such as cobalt (Co). The ceramic provides the thin film coating with its desired hardness at elevated temperatures, while the metal serves to coalesce the particles on the coated surface and to inhibit crack formation. In the plasma spraying process, the particles are injected into a plasma gas jet that heats them to a temperature above the melting point of the metallic casing and melts the casing before the particles impact the surface.

Consider spherical particles comprised of a WC core of diameter $D_i = 16\ \mu m$, which is encased in a Co shell of outer diameter $D_o = 20\ \mu m$. If the particles flow in a plasma gas at $T_\infty = 10{,}000\ K$ and the coefficient associated with convection from the gas to the particles is $h = 20{,}000\ W/m^2 \cdot K$, how long does it take to heat the particles from an initial temperature of $T_i = 300\ K$ to the melting point of cobalt, $T_{mp} = 1770\ K$? The density and specific heat of WC (the core of the particle) are $\rho_c = 16{,}000\ kg/m^3$ and $c_c = 300\ J/kg \cdot K$, while the corresponding values for Co (the outer shell) are $\rho_s = 8900\ kg/m^3$ and $c_s = 750\ J/kg \cdot K$. Once having reached the melting point, how much additional time is required to completely melt the cobalt if its latent heat of fusion is $h_{sf} = 2.59 \times 10^5\ J/kg$? You may use the lumped capacitance method of analysis and neglect radiation exchange between the particle and its surroundings.

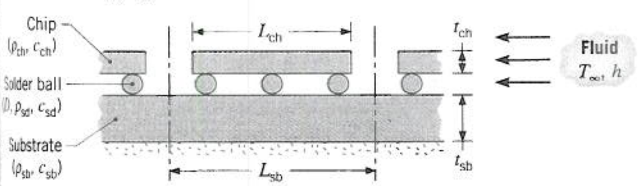
5.27 A chip that is of length $L = 5\ mm$ on a side and thickness $t = 1\ mm$ is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which $h = 150\ W/m^2 \cdot K$ and $T_\infty = 20°C$.



In the off-mode the chip is in thermal equilibrium with the coolant ($T_i = T_\infty$). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with $\dot{q} = 9 \times 10^6$ W/m^3. Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature T_f. Following activation of the chip, how long does it take to come within 1°C of this temperature? The chip density and specific heat are $\rho = 2000$ kg/m^3 and $c = 700$ J/kg · K, respectively.

5.28 Consider the conditions of Problem 5.27. In addition to treating heat transfer by convection directly from the chip to the coolant, a more realistic analysis would account for indirect transfer from the chip to the substrate and then from the substrate to the coolant. The total thermal resistance associated with this indirect route includes contributions due to the chip–substrate interface (a contact resistance), multidimensional conduction in the substrate, and convection from the surface of the substrate to the coolant. If this total thermal resistance is $R_t = 200$ K/W, what is the steady-state chip temperature T_f? Following activation of the chip, how long does it take to come within 1°C of this temperature?

5.29 Thermal stress testing is a common procedure used to assess the reliability of an electronic package. Typically, thermal stresses are induced in soldered or wired connections to reveal mechanisms that could cause failure and must therefore be corrected before the product is released. As an example of the procedure, consider an array of silicon chips ($\rho_{ch} = 2300$ kg/m^3, $c_{ch} = 710$ J/kg · K) joined to an alumina substrate ($\rho_{sb} = 4000$ kg/m^3, $c_{sb} = 770$ J/kg · K) by solder balls ($\rho_{sd} = 11{,}000$ kg/m^3, $c_{sd} = 130$ J/kg · K). Each chip of width L_{ch} and thickness t_{ch} is joined to a unit substrate section of width L_{sb} and thickness t_{sb} by solder balls of diameter D.

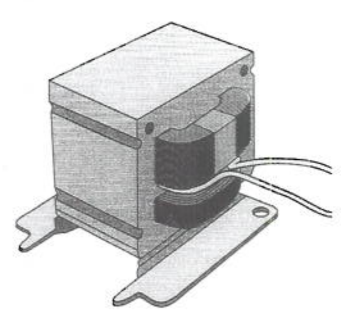


A thermal stress test begins by subjecting the multichip module, which is initially at room temperature, to a hot fluid stream and subsequently cooling the module by exposing it to a cold fluid stream. The process is repeated for a prescribed number of cycles to assess the integrity of the soldered connections.

(a) As a first approximation, assume that there is negligible heat transfer between the components (chip/solder/substrate) of the module and that the thermal response of each component may be determined from a lumped capacitance analysis involving the same convection coefficient h. Assuming no reduction in surface area due to contact between a solder ball and the chip or substrate, obtain expressions for the thermal time constant of each component. Heat transfer is to all surfaces of a chip, but to only the top surface of the substrate. Evaluate the three time constants for $L_{ch} = 15$ mm, $t_{ch} = 2$ mm, $L_{sb} = 25$ mm, $t_{sb} = 10$ mm, $D = 2$ mm, and a value of $h = 50$ W/m^2 · K, which is characteristic of an air stream. Compute and plot the temperature histories of the three components for the heating portion of a cycle, with $T_i = 20$°C and $T_\infty = 80$°C. At what time does each component experience 99% of its maximum possible temperature rise, that is, $(T - T_i)/(T_\infty - T_i) = 0.99$? If the maximum stress on a solder ball corresponds to the maximum difference between its temperature and that of the chip or substrate, when will this maximum occur?

(b) To reduce the time required to complete a stress test, a dielectric liquid could be used in lieu of air to provide a larger convection coefficient of $h = 200$ W/m^2 · K. What is the corresponding savings in time for each component to achieve 99% of its maximum possible temperature rise?

5.30 The objective of this problem is to develop thermal models for estimating the steady-state temperature and the transient temperature history of the electrical transformer shown below.

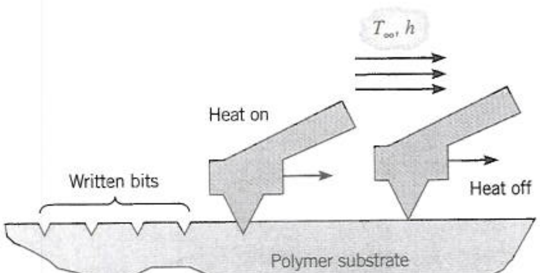


The external transformer geometry is approximately cubical, with a length of 32 mm to a side. The combined mass of the iron and copper in the transformer is 0.28 kg, and its weighted-average specific heat is 400 J/kg · K. The transformer dissipates 4.0 W and is operating in ambient air at $T_\infty = 20°C$, with a convection coefficient of 10 W/m² · K. List and justify the assumptions made in your analysis, and discuss limitations of the models.

(a) Beginning with a properly defined control volume, develop a model for estimating the steady-state temperature of the transformer, $T(\infty)$. Evaluate $T(\infty)$ for the prescribed operating conditions.

(b) Develop a model for estimating the thermal response (temperature history) of the transformer if it is initially at a temperature of $T_i = T_\infty$ and power is suddenly applied. Determine the time required for the transformer to come within 5°C of its steady-state operating temperature.

5.31 In thermomechanical data storage, a processing head, consisting of M heated cantilevers, is used to write data onto an underlying polymer storage medium. Electrical resistance heaters are microfabricated onto each cantilever, which continually travel over the surface of the medium. The resistance heaters are turned on and off by controlling electrical current to each cantilever. As a cantilever goes through a complete heating and cooling cycle, the underlying polymer is softened, and one bit of data is written in the form of a *surface pit* in the polymer. A track of individual data bits (pits), each separated by approximately 50 nm, can be fabricated. Multiple tracks of bits, also separated by approximately 50 nm, are subsequently fabricated into the surface of the storage medium. Consider a single cantilever that is fabricated primarily of silicon with a mass of 50×10^{-18} kg and a surface area of 600×10^{-15} m². The cantilever is initially at $T_i = T_\infty = 300$ K, and the heat transfer coefficient between the cantilever and the ambient is 200×10^3 W/m² · K.



(a) Determine the ohmic heating required to raise the cantilever temperature to $T = 1000$ K within a heating time of $t_h = 1$ μs. *Hint*: See Problem 5.20.

(b) Find the time required to cool the cantilever from 1000 K to 400 K (t_c) and the thermal processing time required for one complete heating and cooling cycle, $t_p = t_h + t_c$.

(c) Determine how many bits (N) can be written onto a 1 mm × 1 mm polymer storage medium. If $M = 100$ cantilevers are ganged onto a single processing head, determine the total thermal processing time needed to write the data.

5.32 The melting of water initially at the fusion temperature, $T_f = 0°C$, was considered in Example 1.5. Freezing of water often occurs at 0°C. However, pure liquids that undergo a cooling process can remain in a *supercooled* liquid state well below their equilibrium freezing temperature, T_f, particularly when the liquid is not in contact with any solid material. Droplets of liquid water in the atmosphere have a supercooled freezing temperature, $T_{f,sc}$, that can be well correlated to the droplet diameter by the expression $T_{f,sc} = -28 + 0.87 \ln(D_p)$ in the diameter range $10^{-7} < D_p < 10^{-2}$ m, where $T_{f,sc}$ has units of degrees Celsius and D_p is expressed in units of meters. For a droplet of diameter $D = 50$ μm and initial temperature $T_i = 10°C$ subject to ambient conditions of $T_\infty = -40°C$ and $h = 900$ W/m² · K, compare the time needed to completely solidify the droplet for case A, when the droplet solidifies at $T_f = 0°C$, and case B, when the droplet starts to freeze at $T_{f,sc}$. Sketch the temperature histories from the initial time to the time when the droplets are completely solid. *Hint:* When the droplet reaches $T_{f,sc}$ in case B, rapid solidification occurs during which the latent energy released by the freezing water is absorbed by the remaining liquid in the drop. As soon as any ice is formed within the droplet, the remaining liquid is in contact with a solid (the ice) and the freezing temperature immediately shifts from $T_{f,sc}$ to $T_f = 0°C$.

5.33 As noted in Problem 5.3, microwave ovens operate by rapidly aligning and reversing water molecules within the food, resulting in volumetric energy generation and, in turn, cooking of the food. When the food is initially frozen, however, the water molecules do not readily oscillate in response to the microwaves, and the volumetric generation rates are between one and two orders of magnitude lower than if the water were in liquid form. (Microwave power that is not absorbed in the food is reflected back to the microwave generator, where it must be dissipated in the form of heat to prevent damage to the generator.)

(a) Consider a frozen, 1-kg spherical piece of ground beef at an initial temperature of $T_i = -20°C$ placed in a microwave oven with $T_\infty = 30°C$ and $h = 15$ W/m² · K. Determine how long it will take the beef to reach a uniform temperature of $T = 0°C$, with all the water in the form of ice. Assume the properties of the beef are the same as ice, and assume 3% of the oven power ($P = 1$ kW total) is absorbed in the food.

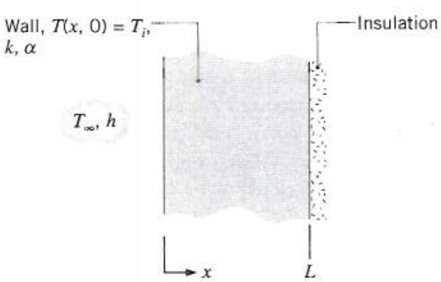
(b) After all the ice is converted to liquid, determine how long it will take to heat the beef to $T_f = 80°C$ if 95% of the oven power is absorbed in the food. Assume the properties of the beef are the same as liquid water.

(c) When thawing food in microwave ovens, one may observe that some of the food may still be frozen while other parts of the food are overcooked. Explain why this occurs. Explain why most microwave ovens have thaw cycles that are associated with very low oven powers.

One-Dimensional Conduction: The Plane Wall

5.34 Consider the series solution, Equation 5.39, for the plane wall with convection. Calculate midplane ($x^* = 0$) and surface ($x^* = 1$) temperatures θ^* for $Fo = 0.1$ and 1, using $Bi = 0.1$, 1, and 10. Consider only the first four eigenvalues. Based on these results discuss the validity of the approximate solutions, Equations 5.40 and 5.41.

5.35 Consider the one-dimensional wall shown in the sketch, which is initially at a uniform temperature T_i and is suddenly subjected to the convection boundary condition with a fluid at T_∞.

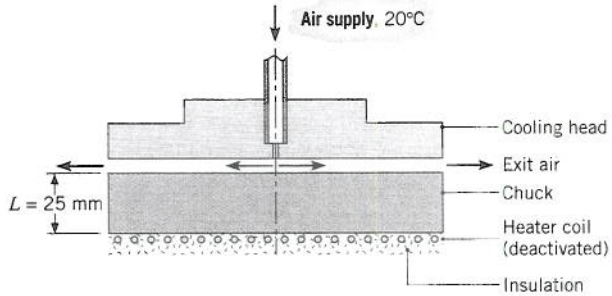


For a particular wall, case 1, the temperature at $x = L_1$ after $t_1 = 100$ s is $T_1(L_1, t_1) = 315°C$. Another wall, case 2, has different thickness and thermal conditions as shown below.

Case	L (m)	α (m^2/s)	k (W/m · K)	T_i (°C)	T_∞ (°C)	h ($W/m^2 \cdot K$)
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

How long will it take for the second wall to reach 28.5°C at the position $x = L_2$? Use as the basis for analysis, the dimensionless functional dependence for the transient temperature distribution expressed in Equation 5.38.

5.36 Referring to the semiconductor processing tool of Problem 5.13, it is desired at some point in the manufacturing cycle to cool the chuck, which is made of aluminum alloy 2024. The proposed cooling scheme passes air at 20°C between the air-supply head and the chuck surface.



(a) If the chuck is initially at a uniform temperature of 100°C, calculate the time required for its lower surface to reach 25°C, assuming a uniform convection coefficient of 50 $W/m^2 \cdot K$ at the head–chuck interface.

(b) Generate a plot of the time-to-cool as a function of the convection coefficient for the range $10 \le h \le 2000$ $W/m^2 \cdot K$. If the lower limit represents a free convection condition without any head present, comment on the effectiveness of the head design as a method for cooling the chuck.

5.37 Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830$ kg/m^3, $c = 550$ J/kg · K, $k = 48$ W/m · K), which is initially at a uniform temperature of $T_i = 200°C$ and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800°C$ maintain a convection coefficient of $h = 250$ $W/m^2 \cdot K$ on both surfaces of the plate. How long should the plate be left in the furnace?

5.38 Consider the heavily insulated pipe of Example 5.4, which is suddenly subjected to the flow of hot oil. Use the *Transient Conduction, Plane Wall* model of *IHT* to obtain the following solutions.

(a) Calculate the temperature of the inner and outer surfaces of the pipe, the heat flux at the inner surface, and the energy transferred to the wall after 8 min. Compare your results with those obtained in the example.

(b) At what time will the outer surface temperature of the pipe, $T(0, t)$, reach 25°C?

(c) Using the *Explore* and *Graph* options of *IHT*, calculate and plot on a single graph the temperature distributions, $T(x, t)$, for the initial condition, the final condition, and intermediate times of 4 and 8 min. Explain key features of the distributions.

(d) Calculate and plot the temperature histories, $T(x, t)$, at the inner ($x = 0$) and outer ($x = L$) pipe surfaces for $0 \leq t \leq 16$ min.

5.39 The 150-mm-thick wall of a gas-fired furnace is constructed of fire-clay brick ($k = 1.5$ W/m · K, $\rho = 2600$ kg/m^3, $c_p = 1000$ J/kg · K) and is well insulated at its outer surface. The wall is at a uniform initial temperature of 20°C, when the burners are fired and the inner surface is exposed to products of combustion for which $T_\infty = 950$°C and $h = 100$ W/m^2 · K.

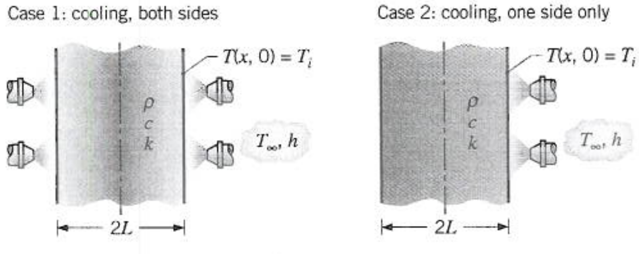
(a) How long does it take for the outer surface of the wall to reach a temperature of 750°C?

(b) Plot the temperature distribution in the wall at the foregoing time, as well as at several intermediate times.

5.40 Steel is sequentially heated and cooled (*annealed*) to relieve stresses and to make it less brittle. Consider a 100-mm-thick plate ($k = 45$ W/m · K, $\rho = 7800$ kg/m^3, $c_p = 500$ J/kg · K) that is initially at a uniform temperature of 300°C and is heated (on both sides) in a gas-fired furnace for which $T_\infty = 700$°C and $h = 500$ W/m^2 · K. How long will it take for a minimum temperature of 550°C to be reached in the plate?

5.41 A plate of thickness $2L = 25$ mm at a temperature of 600°C is removed from a hot pressing operation and must be cooled rapidly in order to achieve the required physical properties. The process engineer plans to use air jets to control the rate of cooling, but she is uncertain whether it is necessary to cool both sides (case 1) or only one side (case 2) of the plate. The concern is not just for the *time-to-cool*, but also for the maximum temperature difference within the plate. If this temperature difference is too large, the plate can experience significant warping.

Case 1: cooling, both sides   Case 2: cooling, one side only



The air supply is at 25°C, and the convection coefficient on the surface is 400 W/m^2 · K. The thermophysical properties of the plate are $\rho = 3000$ kg/m^3, $c = 750$ J/kg · K, and $k = 15$ W/m · K.

(a) Using the *IHT* software, calculate and plot on one graph the temperature histories for cases 1 and 2 for a 500-s cooling period. Compare the times required for the maximum temperature in the plate to reach 100°C. Assume no heat loss from the unexposed surface of case 2.

(b) For both cases, calculate and plot on one graph the variation with time of the maximum temperature difference in the plate. Comment on the relative magnitudes of the temperature gradients within the plate as a function of time.

5.42 During transient operation, the steel nozzle of a rocket engine must not exceed a maximum allowable operating temperature of 1500 K when exposed to combustion gases characterized by a temperature of 2300 K and a convection coefficient of 5000 W/m^2 · K. To extend the duration of engine operation, it is proposed that a ceramic *thermal barrier coating* ($k = 10$ W/m · K, $\alpha = 6 \times 10^{-6}$ m^2/s) be applied to the interior surface of the nozzle.

(a) If the ceramic coating is 10 mm thick and at an initial temperature of 300 K, obtain a conservative estimate of the maximum allowable duration of engine operation. The nozzle radius is much larger than the combined wall and coating thickness.

(b) Compute and plot the inner and outer surface temperatures of the coating as a function of time for $0 \leq t \leq 150$ s. Repeat the calculations for a coating thickness of 40 mm.

5.43 In a tempering process, glass plate, which is initially at a uniform temperature T_i, is cooled by suddenly reducing the temperature of both surfaces to T_s. The plate is 20 mm thick, and the glass has a thermal diffusivity of 6×10^{-7} m^2/s.

(a) How long will it take for the midplane temperature to achieve 50% of its maximum possible temperature reduction?

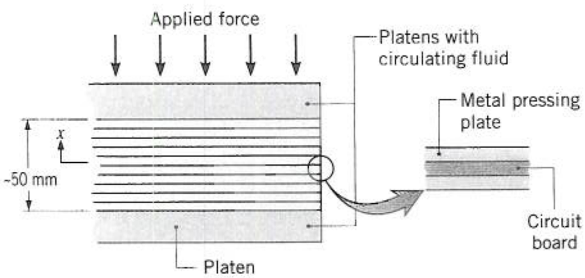
(b) If $(T_i - T_s) = 300$°C, what is the maximum temperature gradient in the glass at the above time?

5.44 The strength and stability of tires may be enhanced by heating both sides of the rubber ($k = 0.14$ W/m · K, $\alpha = 6.35 \times 10^{-8}$ m^2/s) in a steam chamber for which $T_\infty = 200$°C. In the heating process, a 20-mm-thick rubber wall (assumed to be untreaded) is taken from an initial temperature of 25°C to a midplane temperature of 150°C.

(a) If steam flow over the tire surfaces maintains a convection coefficient of $h = 200$ W/m^2 · K, how long will it take to achieve the desired midplane temperature?

(b) To accelerate the heating process, it is recommended that the steam flow be made sufficiently vigorous to maintain the tire surfaces at 200°C throughout the process. Compute and plot the midplane and surface temperatures for this case, as well as for the conditions of part (a).

5.45 Copper-coated, epoxy-filled fiberglass circuit boards are treated by heating a stack of them under high pressure as shown in the sketch. The purpose of the pressing–heating operation is to cure the epoxy that bonds the fiberglass sheets, imparting stiffness to the boards. The stack, referred to as a *book*, is comprised of 10 *boards* and 11 *pressing plates*, which prevent epoxy from flowing between the boards and impart a smooth finish to the cured boards. In order to perform simplified thermal analyses, it is reasonable to approximate the book as having an effective thermal conductivity (k) and an effective thermal capacitance (ρc_p). Calculate the effective properties if each of the boards and plates has a thickness of 2.36 mm and the following thermophysical properties: board (b) $\rho_b = 1000$ kg/m^3, $c_{p,b} = 1500$ J/kg · K, $k_b = 0.30$ W/m · K; plate (p) $\rho_p = 8000$ kg/m^3, $c_{p,p} = 480$ J/kg · K, $k_p = 12$ W/m · K.

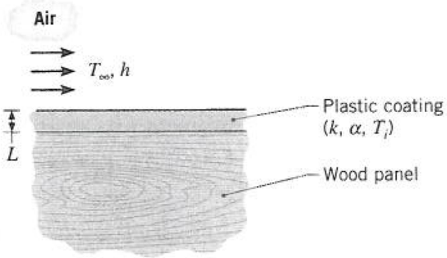


5.46 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.45. The platens at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. The purpose of the pressing–heating operation is to cure the epoxy, which bonds the fiberglass sheets, and impart stiffness to the boards. The cure condition is achieved when the epoxy has been maintained at or above 170°C for at least 5 min. The effective thermophysical properties of the stack or *book* (boards and metal pressing plates) are $k = 0.613$ W/m · K and $\rho c_p = 2.73 \times 10^6$ J/m^3 · K.

(a) If the book is initially at 15°C and, following application of pressure, the platens are suddenly brought to a uniform temperature of 190°C, calculate the elapsed time t_e required for the midplane of the book to reach the cure temperature of 170°C.

(b) If, at this instant of time, $t = t_e$, the platen temperature were reduced suddenly to 15°C, how much energy would have to be removed from the book by the coolant circulating in the platen, in order to return the stack to its initial uniform temperature?

5.47 A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at 25°C. As first approximations, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic.



If the thickness of the coating is $L = 2$ mm and it has an initial uniform temperature of $T_i = 200$°C, how long will it take for the surface to achieve a *safe-to-touch* temperature of 42°C if the convection coefficient is $h = 200$ W/m^2 · K? What is the corresponding value of the interface temperature? The thermal conductivity and diffusivity of the plastic are $k = 0.25$ W/m · K and $\alpha = 1.20 \times 10^{-7}$ m^2/s, respectively.

One-Dimensional Conduction: The Long Cylinder

5.48 A long rod of 60-mm diameter and thermophysical properties $\rho = 8000$ kg/m^3, $c = 500$ J/kg · K, and $k = 50$ W/m · K is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/m^2 · K.

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

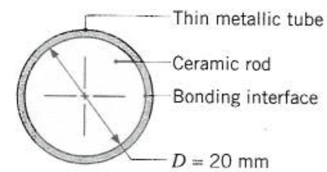
(b) In a heat-treating process, the centerline temperature of the rod must be increased from $T_i = 300$ K to $T = 500$ K. Compute and plot the centerline temperature histories for $h = 100$, 500, and 1000 W/m^2 · K. In each case the calculation may be terminated when $T = 500$ K.

5.49 A long cylinder of 30-mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. The cylinder properties are $k = 1.7$ W/m · K, $c = 1600$ J/kg · K, and $\rho = 400$ kg/m^3, while the convection coefficient is 50 W/m^2 · K.

(a) Calculate the time required for the surface of the cylinder to reach 500 K.

(b) Compute and plot the surface temperature history for $0 \le t \le 300$ s. If the oil were agitated, providing a convection coefficient of 250 $W/m^2 \cdot K$, how would the temperature history change?

5.50 A long pyroceram rod of diameter 20 mm is clad with a very thin metallic tube for mechanical protection. The bonding between the rod and the tube has a thermal contact resistance of $R'_{t,c} = 0.12$ m · K/W.



(a) If the rod is initially at a uniform temperature of 900 K and is suddenly cooled by exposure to an airstream for which $T_\infty = 300$ K and $h = 100$ $W/m^2 \cdot K$, at which time will the centerline reach 600 K?

(b) Cooling may be accelerated by increasing the airspeed and hence the convection coefficient. For values of $h = 100$, 500, and 1000 $W/m^2 \cdot K$, compute and plot the centerline and surface temperatures of the pyroceram as a function of time for $0 \le t \le 300$ s. Comment on the implications of achieving enhanced cooling solely by increasing h.

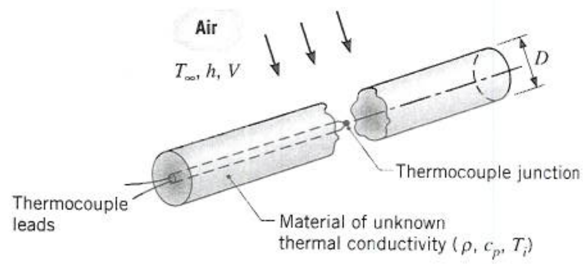
5.51 A long rod 40 mm in diameter, fabricated from sapphire (aluminum oxide) and initially at a uniform temperature of 800 K, is suddenly cooled by a fluid at 300 K having a heat transfer coefficient of 1600 $W/m^2 \cdot K$. After 35 s, the rod is wrapped in insulation and experiences no heat losses. What will be the temperature of the rod after a long period of time?

5.52 A long plastic rod of 30-mm diameter ($k = 0.3$ W/m · K and $\rho c_p = 1040$ $kJ/m^3 \cdot K$) is uniformly heated in an oven as preparation for a pressing operation. For best results, the temperature in the rod should not be less than 200°C. To what uniform temperature should the rod be heated in the oven if, for the worst case, the rod sits on a conveyor for 3 min while exposed to convection cooling with ambient air at 25°C and with a convection coefficient of 8 $W/m^2 \cdot K$? A further condition for good results is a maximum–minimum temperature difference of less than 10°C. Is this condition satisfied and, if not, what could you do to satisfy it?

5.53 As part of a heat treatment process, cylindrical, 304 stainless steel rods of 100-mm diameter are cooled from an initial temperature of 500°C by suspending them in an oil bath at 30°C. If a convection coefficient of 500 $W/m^2 \cdot K$ is maintained by circulation of the oil, how long does it take for the centerline of a rod to reach a temperature of 50°C, at which point it is withdrawn from the bath? If 10 rods of length $L = 1$ m are processed per hour, what is the nominal rate at which energy must be extracted from the bath (the cooling load)?

5.54 In a manufacturing process, long rods of different diameters are at a uniform temperature of 400°C in a curing oven, from which they are removed and cooled by forced convection in air at 25°C. One of the line operators has observed that it takes 280 seconds for a 40-mm-diameter rod to cool to an *safe-to-handle* temperature of 60°C. For an equivalent convection coefficient, how long will it take for an 80-mm-diameter rod to cool to the same temperature? The thermophysical properties of the rod are $\rho = 2500$ kg/m^3, $c = 900$ J/kg · K, and $k = 15$ W/m · K. Comment on your result. Did you anticipate this outcome?

5.55 The density and specific heat of a particular material are known ($\rho = 1200$ kg/m^3, $c_p = 1250$ J/kg · K), but its thermal conductivity is unknown. To determine the thermal conductivity, a long cylindrical specimen of diameter $D = 40$ mm is machined, and a thermocouple is inserted through a small hole drilled along the centerline.



The thermal conductivity is determined by performing an experiment in which the specimen is heated to a uniform temperature of $T_i = 100$°C and then cooled by passing air at $T_\infty = 25$°C in cross flow over the cylinder. For the prescribed air velocity, the convection coefficient is $h = 55$ $W/m^2 \cdot K$.

(a) If a centerline temperature of $T(0, t) = 40$°C is recorded after $t = 1136$ s of cooling, verify that the material has a thermal conductivity of $k = 0.30$ W/m · K.

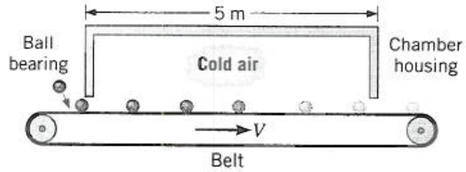
(b) For air in cross-flow over the cylinder, the prescribed value of $h = 55$ $W/m^2 \cdot K$ corresponds to a velocity of $V = 6.8$ m/s. If $h = CV^{0.618}$, where the constant C has units of $W \cdot s^{0.618}/m^{2.618} \cdot K$, how does the centerline temperature at $t = 1136$ s vary with velocity for $3 \le V \le 20$ m/s? Determine the centerline temperature histories for $0 \le t \le 1500$ s and velocities of 3, 10, and 20 m/s.

5.56 In Section 5.2 we noted that the value of the Biot number significantly influences the nature of the temperature distribution in a solid during a transient conduction process. Reinforce your understanding of this important concept by using the *IHT* model for one-dimensional transient conduction to determine radial temperature distributions in a 30-mm-diameter, stainless steel rod ($k = 15$ W/m · K, $\rho = 8000$ kg/m^3, $c_p = 475$ J/kg · K), as it is cooled from an initial uniform temperature of 325°C by a fluid at 25°C. For the following values of the convection coefficient and the designated times, determine the radial temperature distribution: $h = 100$ W/m^2 · K ($t = 0$, 100, 500 s); $h = 1000$ W/m^2 · K ($t = 0$, 10, 50 s); $h = 5000$ W/m^2 · K ($t = 0$, 1, 5, 25 s). Prepare a separate graph for each convection coefficient, on which temperature is plotted as a function of dimensionless radius at the designated times.

One-Dimensional Conduction: The Sphere

5.57 In heat treating to harden steel ball bearings ($c = 500$ J/kg · K, $\rho = 7800$ kg/m^3, $k = 50$ W/m · K), it is desirable to increase the surface temperature for a short time without significantly warming the interior of the ball. This type of heating can be accomplished by sudden immersion of the ball in a molten salt bath with $T_\infty = 1300$ K and $h = 5000$ W/m^2 · K. Assume that any location within the ball whose temperature exceeds 1000 K will be hardened. Estimate the time required to harden the outer millimeter of a ball of diameter 20 mm, if its initial temperature is 300 K.

5.58 A cold air chamber is proposed for quenching steel ball bearings of diameter $D = 0.2$ m and initial temperature $T_i = 400$°C. Air in the chamber is maintained at −15°C by a refrigeration system, and the steel balls pass through the chamber on a conveyor belt. Optimum bearing production requires that 70% of the initial thermal energy content of the ball above −15°C be removed. Radiation effects may be neglected, and the convection heat transfer coefficient within the chamber is 1000 W/m^2 · K. Estimate the residence time of the balls within the chamber, and recommend a drive velocity of the conveyor. The following properties may be used for the steel: $k = 50$ W/m · K, $\alpha = 2 \times 10^{-5}$ m^2/s, and $c = 450$ J/kg · K.



5.59 Stainless steel (AISI 304) ball bearings, which have uniformly been heated to 850°C, are hardened by quenching them in an oil bath that is maintained at 40°C. The ball diameter is 20 mm, and the convection coefficient associated with the oil bath is 1000 W/m^2 · K.

(a) If quenching is to occur until the surface temperature of the balls reaches 100°C, how long must the balls be kept in the oil? What is the center temperature at the conclusion of the cooling period?

(b) If 10,000 balls are to be quenched per hour, what is the rate at which energy must be removed by the oil bath cooling system in order to maintain its temperature at 40°C?

5.60 A sphere 30 mm in diameter initially at 800 K is quenched in a large bath having a constant temperature of 320 K with a convection heat transfer coefficient of 75 W/m^2 · K. The thermophysical properties of the sphere material are: $\rho = 400$ kg/m^3, $c = 1600$ J/kg · K, and $k = 1.7$ W/m · K.

(a) Show, in a qualitative manner on T–t coordinates, the temperatures at the center and at the surface of the sphere as a function of time.

(b) Calculate the time required for the surface of the sphere to reach 415 K.

(c) Determine the heat flux (W/m^2) at the outer surface of the sphere at the time determined in part (b).

(d) Determine the energy (J) that has been lost by the sphere during the process of cooling to the surface temperature of 415 K.

(e) At the time determined by part (b), the sphere is quickly removed from the bath and covered with perfect insulation, such that there is no heat loss from the surface of the sphere. What will be the temperature of the sphere after a long period of time has elapsed?

(f) Compute and plot the center and surface temperature histories over the period $0 \le t \le 150$ s. What effect does an increase in the convection coefficient to $h = 200$ W/m^2 · K have on the foregoing temperature histories? For $h = 75$ and 200 W/m^2 · K, compute and plot the surface heat flux as a function of time for $0 \le t \le 150$ s.

5.61 Spheres A and B are initially at 800 K, and they are simultaneously quenched in large constant temperature baths, each having a temperature of 320 K. The following parameters are associated with each of the spheres and their cooling processes.

	Sphere A	Sphere B
Diameter (mm)	300	30
Density (kg/m^3)	1600	400
Specific heat ($kJ/kg \cdot K$)	0.400	1.60
Thermal conductivity ($W/m \cdot K$)	170	1.70
Convection coefficient ($W/m^2 \cdot K$)	5	50

(a) Show in a qualitative manner, on T versus t coordinates, the temperatures at the center and at the surface for each sphere as a function of time. Briefly explain the reasoning by which you determine the relative positions of the curves.

(b) Calculate the time required for the surface of each sphere to reach 415 K.

(c) Determine the energy that has been gained by each of the baths during the process of the spheres cooling to 415 K.

5.62 Spheres of 40-mm diameter heated to a uniform temperature of 400°C are suddenly removed from the oven and placed in a forced-air bath operating at 25°C with a convection coefficient of 300 $W/m^2 \cdot K$ on the sphere surfaces. The thermophysical properties of the sphere material are $\rho = 3000\ kg/m^3$, $c = 850\ J/kg \cdot K$, and $k = 15\ W/m \cdot K$.

(a) How long must the spheres remain in the air bath for 80% of the thermal energy to be removed?

(b) The spheres are then placed in a packing carton that prevents further heat transfer to the environment. What uniform temperature will the spheres eventually reach?

5.63 Consider the packed bed operating conditions of Problem 5.12, but with Pyrex ($\rho = 2225\ kg/m^3$, $c = 835\ J/kg \cdot K$, $k = 1.4\ W/m \cdot K$) used instead of aluminum. How long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere?

5.64 The convection coefficient for flow over a solid sphere may be determined by submerging the sphere, which is initially at 25°C, into the flow, which is at 75°C, and measuring its surface temperature at some time during the transient heating process.

(a) If the sphere has a diameter of 0.1 m, a thermal conductivity of 15 $W/m \cdot K$, and a thermal diffusivity of $10^{-5}\ m^2/s$, at what time will a surface temperature of 60°C be recorded if the convection coefficient is 300 $W/m^2 \cdot K$?

(b) Assess the effect of thermal diffusivity on the thermal response of the material by computing center and surface temperature histories for $\alpha = 10^{-6}$, 10^{-5}, and $10^{-4}\ m^2/s$. Plot your results for the period $0 \le t \le 300$ s. In a similar manner, assess the effect of thermal conductivity by considering values of $k = 1.5$, 15, and 150 $W/m \cdot K$.

5.65 Consider the sphere of Example 5.5, which is initially at a uniform temperature when it is suddenly removed from the furnace and subjected to a two-step cooling process. Use the *Transient Conduction, Sphere* model of *IHT* to obtain the following solutions.

(a) For step 1, calculate the time required for the center temperature to reach $T(0, t) = 335°C$, while cooling in air at 20°C with a convection coefficient of 10 $W/m^2 \cdot K$. What is the Biot number for this cooling process? Do you expect radial temperature gradients to be appreciable? Compare your results to those of the example.

(b) For step 2, calculate the time required for the center temperature to reach $T(0, t) = 50°C$, while cooling in a water bath at 20°C with a convection coefficient of 6000 $W/m^2 \cdot K$.

(c) For the step 2 cooling process, calculate and plot the temperature histories, $T(r, t)$, for the center and surface of the sphere. Identify and explain key features of the histories. When do you expect the temperature gradients in the sphere to be the largest?

Semi-Infinite Media

5.66 Two large blocks of different materials, such as copper and concrete, have been sitting in a room (23°C) for a very long time. Which of the two blocks, if either, will feel colder to the touch? Assume the blocks to be semi-infinite solids and your hand to be at a temperature of 37°C.

5.67 A plane wall of thickness 0.6 m ($L = 0.3$ m) is made of steel ($k = 30\ W/m \cdot K$, $\rho = 7900\ kg/m^3$, $c = 640\ J/kg \cdot K$). It is initially at a uniform temperature and is then exposed to air on both surfaces. Consider two different convection conditions: natural convection, characterized by $h = 10\ W/m^2 \cdot K$, and forced convection, with $h = 100\ W/m^2 \cdot K$. You are to calculate the surface temperature at three different times—$t = 2.5$ min, 25 min, and 250 min—for a total of six different cases.

(a) For each of these six cases, calculate the nondimensional surface temperature, $\theta_s^* = (T_s - T_\infty)/(T_i - T_\infty)$, using four different methods: exact solution, first-term-of-the-series solution, lumped capacitance, and semi-infinite solid. Present your results in a table.

(b) Briefly explain the conditions for which (i) the first-term solution is a good approximation to the exact solution, (ii) the lumped capacitance solution is a good approximation, (iii) the semi-infinite solid solution is a good approximation.

5.68 Asphalt pavement may achieve temperatures as high as 50°C on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to 20°C. Calculate the total amount of energy (J/m^2) that will be transferred from the asphalt over a 30-min period in which the surface is maintained at 20°C.

5.69 A thick steel slab ($\rho = 7800$ kg/m^3, $c = 480$ J/kg · K, $k = 50$ W/m · K) is initially at 300°C and is cooled by water jets impinging on one of its surfaces. The temperature of the water is 25°C, and the jets maintain an extremely large, approximately uniform convection coefficient at the surface. Assuming that the surface is maintained at the temperature of the water throughout the cooling, how long will it take for the temperature to reach 50°C at a distance of 25 mm from the surface?

5.70 Consider the water main of Example 5.6, which is buried in soil initially at 20°C and is suddenly subjected to a constant surface temperature of −15°C for 60 days. Use the *Transient Conduction/Semi-Infinite Solid* model of *IHT* to obtain the following solutions. Compare your results with those in the Comments section of the example.

(a) Calculate and plot the temperature history at the burial depth of 0.68 m for thermal diffusivities of $\alpha \times 10^7 = 1.0$, 1.38, and 3.0 m^2/s.

(b) For $\alpha = 1.38 \times 10^{-7}$ m^2/s, plot the temperature distribution over the depth $0 \le x \le 1.0$ m for times of 1, 5, 10, 30, and 60 days.

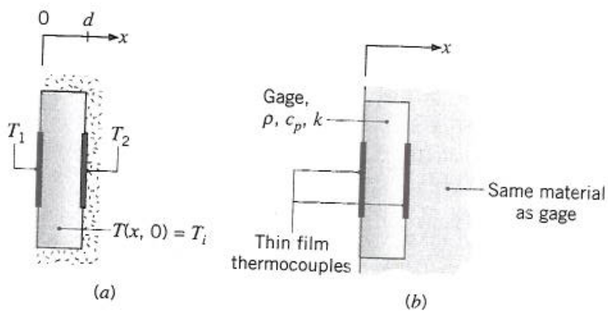
(c) For $\alpha = 1.38 \times 10^{-7}$ m^2/s, show that the heat flux from the soil decreases with increasing time by plotting $q''_x(0, t)$ as a function of time for the 60-day period. On this graph, also plot the heat flux at the depth of the buried main, $q''_x(0.68 \text{ m}, t)$.

5.71 A tile-iron consists of a massive plate maintained at 150°C by an imbedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50°C for at least 2 min, but its temperature should not exceed 120°C to avoid deterioration of the adhesive. Assume the tile and subfloor to have an initial temperature of 25°C and to have equivalent thermophysical properties of $k = 0.15$ W/m · K and $\rho c_p = 1.5 \times 10^6$ J/m^3 · K.

(a) How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120°C?

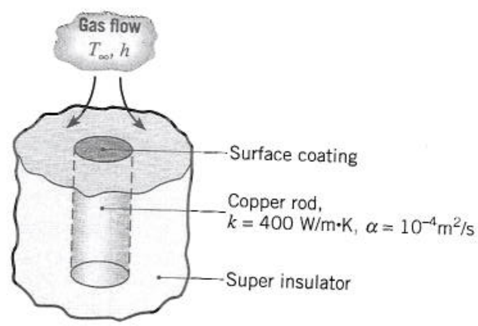
(b) If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift the tile?

5.72 The manufacturer of a heat flux gage like that illustrated in Problem 1.12 claims the time constant for a 63.2% response to be $\tau = (4d^2\rho c_p)/\pi^2 k$, where ρ, c_p, and k are the thermophysical properties of the gage material and d is its thickness. Not knowing the origin of this relation, your task is to model the gage considering the two extreme cases illustrated below. In both cases, the gage, initially at a uniform temperature T_i, is exposed to a sudden change in surface temperature, $T(0, t) = T_s$. For case (*a*) the backside of the gage is insulated, and for case (*b*) the gage is imbedded in a semi-infinite solid having the same thermophysical properties as those of the gage.



Develop relationships for predicting the time constant of the gage for the two cases and compare them to the manufacturer's relation. What conclusion can you draw from this analysis regarding the transient response of gages for different applications?

5.73 A simple procedure for measuring surface convection heat transfer coefficients involves coating the surface with a thin layer of material having a precise melting point temperature. The surface is then heated and, by determining the time required for melting to occur, the



convection coefficient is determined. The following experimental arrangement uses the procedure to determine the convection coefficient for gas flow normal to a surface. Specifically, a long copper rod is encased in a super insulator of very low thermal conductivity, and a very thin coating is applied to its exposed surface.

If the rod is initially at 25°C and gas flow for which $h = 200\ \mathrm{W/m^2 \cdot K}$ and $T_\infty = 300°\mathrm{C}$ is initiated, what is the melting point temperature of the coating if melting is observed to occur at $t = 400$ s?

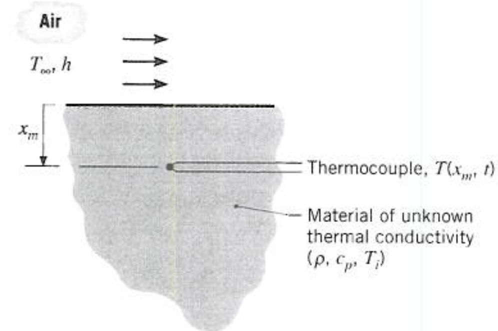
5.74 An insurance company has hired you as a consultant to improve their understanding of burn injuries. They are especially interested in injuries induced when a portion of a worker's body comes into contact with machinery that is at elevated temperatures in the range of 50 to 100°C. Their medical consultant informs them that irreversible thermal injury (cell death) will occur in any living tissue that is maintained at $T \geq 48°\mathrm{C}$ for a duration $\Delta t \geq 10$ s. They want information concerning the extent of irreversible tissue damage (as measured by distance from the skin surface) as a function of the machinery temperature and the time during which contact is made between the skin and the machinery. Assume that living tissue has a normal temperature of 37°C, is isotropic, and has constant properties equivalent to those of liquid water.

(a) To assess the seriousness of the problem, compute locations in the tissue at which the temperature will reach 48°C after 10 s of exposure to machinery at 50°C and 100°C.

(b) For a machinery temperature of 100°C and $0 \leq t \leq 30$ s, compute and plot temperature histories at tissue locations of 0.5, 1, and 2 mm from the skin.

5.75 A procedure for determining the thermal conductivity of a solid material involves embedding a thermocouple in a thick slab of the solid and measuring the response to a prescribed change in temperature at one surface. Consider an arrangement for which the thermocouple is embedded 10 mm from a surface that is suddenly brought to a temperature of 100°C by exposure to boiling water. If the initial temperature of the slab was 30°C and the thermocouple measures a temperature of 65°C, 2 min after the surface is brought to 100°C, what is its thermal conductivity? The density and specific heat of the solid are known to be 2200 kg/m^3 and 700 J/kg · K.

5.76 The density and specific heat of a plastic material are known ($\rho = 950\ \mathrm{kg/m^3}$, $c_p = 1100\ \mathrm{J/kg \cdot K}$), but its thermal conductivity is unknown. To determine the thermal conductivity, an experiment is performed in which a thick sample of the material is heated to a uniform temperature of 100°C and then cooled by passing air at 25°C over one surface. A thermocouple embedded a distance of $x_m = 10$ mm below the surface records the thermal response of the plastic during cooling.



If the convection coefficient associated with the air flow is $h = 200\ \mathrm{W/m^2 \cdot K}$ and a temperature of 60°C is recorded 5 min after the onset of cooling, what is the thermal conductivity of the material?

5.77 A very thick slab with thermal diffusivity $5.6 \times 10^{-6}\ \mathrm{m^2/s}$ and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is 100 W/m^2 · K.

(a) Determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

(b) Compute and plot temperature histories ($0 \leq t \leq 300$ s) at $x = 0$ and $x = 45$ mm for the following parametric variations: (i) $\alpha = 5.6 \times 10^{-7}$, 5.6×10^{-6}, and $5.6 \times 10^{-5}\ \mathrm{m^2/s}$; and (ii) $k = 2$, 20, and 200 W/m · K.

5.78 A thick oak wall, initially at 25°C, is suddenly exposed to combustion products for which $T_\infty = 800°\mathrm{C}$ and $h = 20\ \mathrm{W/m^2 \cdot K}$.

(a) Determine the time of exposure required for the surface to reach the ignition temperature of 400°C.

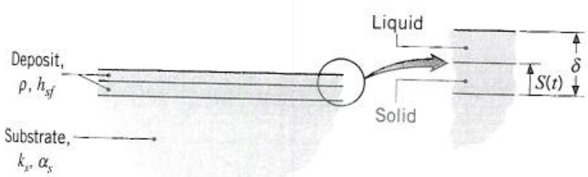
(b) Plot the temperature distribution $T(x)$ in the medium at $t = 325$ s. The distribution should extend to a location for which $T \approx 25°\mathrm{C}$.

5.79 Standards for firewalls may be based on their thermal response to a prescribed radiant heat flux. Consider a 0.25-m-thick concrete wall ($\rho = 2300\ \mathrm{kg/m^3}$, $c = 880\ \mathrm{J/kg \cdot K}$, $k = 1.4\ \mathrm{W/m \cdot K}$), which is at an initial temperature of $T_i = 25°\mathrm{C}$ and irradiated at one surface by lamps that provide a uniform heat flux of $q''_s = 10^4\ \mathrm{W/m^2}$. The absorptivity of the surface to the irradiation is $\alpha_s = 1.0$. If building code requirements dictate that the temperatures of the irradiated and back surfaces must not exceed 325°C and 25°C, respectively, after 30 min of heating, will the requirements be met?

5.80 It is well known that, although two materials are at the same temperature, one may feel cooler to the touch than the other. Consider thick plates of copper and glass, each at an initial temperature of 300 K. Assuming your finger to be at an initial temperature of 310 K and to have thermophysical properties of $\rho = 1000\ \text{kg/m}^3$, $c = 4180\ \text{J/kg}\cdot\text{K}$, and $k = 0.625\ \text{W/m}\cdot\text{K}$, determine whether the copper or the glass will feel cooler to the touch.

5.81 Two stainless steel plates ($\rho = 8000\ \text{kg/m}^3$, $c = 500\ \text{J/kg}\cdot\text{K}$, $k = 15\ \text{W/m}\cdot\text{K}$), each 20 mm thick and insulated on one surface, are initially at 400 and 300 K when they are pressed together at their uninsulated surfaces. What is the temperature of the insulated surface of the hot plate after 1 min has elapsed?

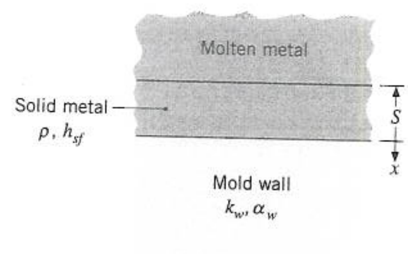
5.82 Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness S of the solid layer becomes equal to the thickness δ of the deposit.



(a) Consider conditions for which molten material at its fusion temperature T_f is deposited on a *large* substrate that is at an initial uniform temperature T_i. With $S = 0$ at $t = 0$, develop an expression for estimating the time t_d required to completely solidify the deposit if it remains at T_f throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (k_s, α_s), the density and latent heat of fusion of the deposit (ρ, h_{sf}), the deposit thickness δ, and the relevant temperatures (T_f, T_i).

(b) The plasma spray deposition process of Problem 5.25 is used to apply a thin ($\delta = 2$ mm) alumina coating on a thick tungsten substrate. The substrate has a uniform initial temperature of $T_i = 300$ K, and its thermal conductivity and thermal diffusivity may be approximated as $k_s = 120\ \text{W/m}\cdot\text{K}$ and $\alpha_s = 4.0 \times 10^{-5}\ \text{m}^2/\text{s}$, respectively. The density and latent heat of fusion of the alumina are $\rho = 3970\ \text{kg/m}^3$ and $h_{sf} = 3577$ kJ/kg, respectively, and the alumina solidifies at its fusion temperature ($T_f = 2318$ K). Assuming that the molten layer is instantaneously deposited on the substrate, estimate the time required for the deposit to solidify.

5.83 When a molten metal is cast in a mold that is a poor conductor, the dominant resistance to heat flow is within the mold wall. Consider conditions for which a liquid metal is solidifying in a thick-walled mold of thermal conductivity k_w and thermal diffusivity α_w. The density and latent heat of fusion of the metal are designated as ρ and h_{sf}, respectively, and in both its molten and solid states, the thermal conductivity of the metal is very much larger than that of the mold.

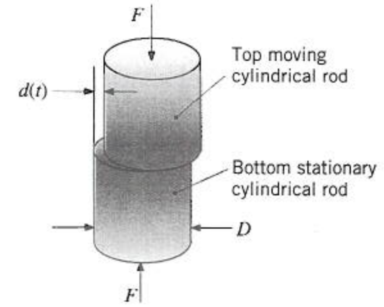


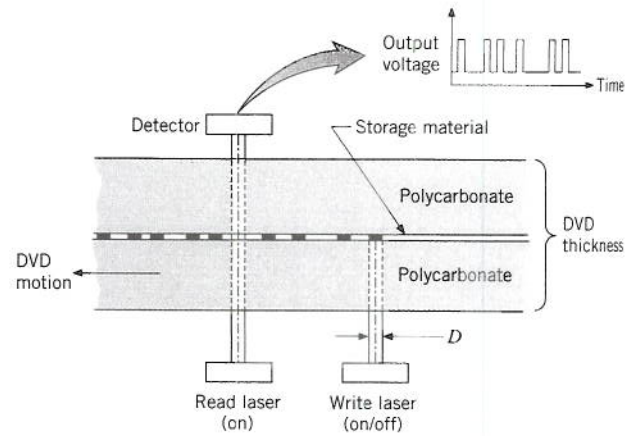
Just before the start of solidification ($S = 0$), the mold wall is everywhere at an initial uniform temperature T_i and the molten metal is everywhere at its fusion (melting point) temperature of T_f. Following the start of solidification, there is conduction heat transfer into the mold wall and the thickness of the solidified metal, S, increases with time t.

(a) Sketch the one-dimensional temperature distribution, $T(x)$, in the mold wall and the metal at $t = 0$ and at two subsequent times during the solidification. Clearly indicate any underlying assumptions.

(b) Obtain a relation for the variation of the solid layer thickness S with time t, expressing your result in terms of appropriate parameters of the system.

5.84 Joints of high quality can be formed by friction welding. Consider the friction welding of two 40-mm-diameter Inconel rods. The bottom rod is stationary, while the top rod is forced into a back-and-forth linear motion characterized by an instantaneous horizontal displacement, $d(t) = a\cos(\omega t)$ where $a = 2$ mm and $\omega = 1000$ rad/s. The coefficient of sliding friction between the two pieces is $\mu = 0.3$. Determine the compressive force that must be applied in order to heat the joint to the Inconel melting point within $t = 3$ s, starting from an initial temperature of 20°C. *Hint*: The frequency of the motion and resulting heat rate are very high. The temperature response can be approximated as if the heating rate were constant in time, equal to its average value.





Objects with Constant Surface Temperatures or Surface Heat Fluxes and Periodic Heating

5.85 An above-ground circular swimming pool is heated so that it can be used in cool weather. The ground has a temperature of 10°C far from the pool. The heater is turned on and quickly brings the pool water to a comfortable 20°C; assume this is the temperature of the circular region of the ground beneath the swimming pool. The pool has a diameter of 5 m.

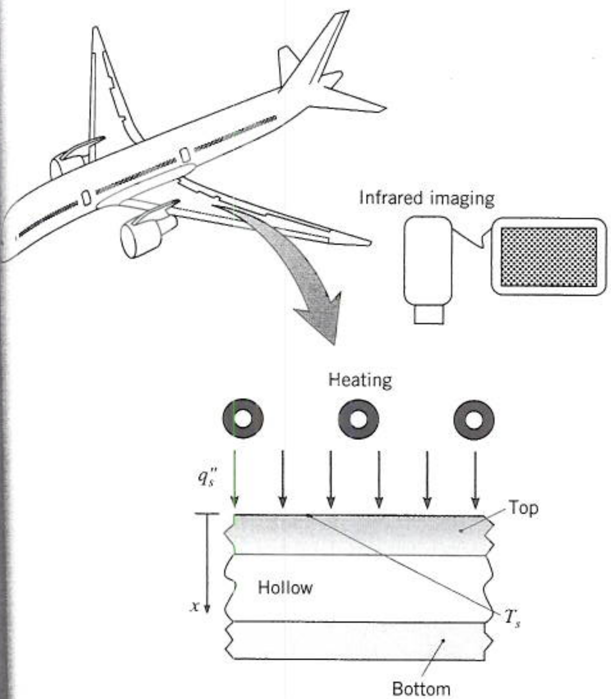
(a) Calculate the rate of heat transfer from the pool to the ground, ten hours after the heater is turned on. *Hint:* Based on symmetry considerations, the pool footprint can be viewed as a heated disk in *infinite* surroundings.

(b) Calculate the time it would take for the heat transfer rate to reach within 10% of its steady-state value.

5.86 A rewritable optical disc (DVD) is formed by sandwiching a 15-nm-thick binary compound storage material between two 1-mm-thick polycarbonate sheets. Data are *written* to the opaque storage medium by irradiating it from below with a relatively high-powered laser beam of diameter 0.4 μm and power 1 mW, resulting in rapid heating of the compound material (the polycarbonate is transparent to the laser irradiation). If the temperature of the storage medium exceeds 900 K, a noncrystalline, amorphous material forms at the heated spot when the laser irradiation is curtailed and the spot is allowed to cool rapidly. The resulting spots of amorphous material have a different reflectivity from the surrounding crystalline material, so they can subsequently be *read* by irradiating them with a second, low-power laser and detecting the changes in laser radiation transmitted through the entire DVD thickness. Determine the irradiation (write) time needed to raise the storage medium temperature from an initial value of 300 K to 1000 K. The absorptivity of the storage medium is 0.8. The polycarbonate properties are ρ = 1200 kg/m^3, k = 0.21 W/m · K, and c_p = 1260 J/kg · K.

5.87 To enable cooking a wider range of foods in microwave ovens, thin, metallic packaging materials have been developed that will readily absorb microwave energy. As the packaging material is heated by the microwaves, conduction simultaneously occurs from the hot packaging material to the cold food. Consider the spherical piece of frozen ground beef of Problem 5.33 that is now wrapped in the thin microwave-absorbing packaging material. Determine the time needed for the beef that is immediately adjacent to the packaging material to reach T = 0°C if 50% of the oven power (P = 1 kW total) is absorbed in the packaging material.

5.88 The structural components of modern aircraft are commonly fabricated of high-performance composite materials. These materials are fabricated by impregnating mats of extremely strong fibers that are held within a form with an epoxy or thermoplastic liquid. After the liquid cures or cools, the resulting component is of extremely high strength and low weight. Periodically, these components must be inspected to ensure that the fiber mats and bonding material do not become delaminated and, in turn, the component loses its airworthiness. One inspection method involves application of a uniform, constant radiation heat flux to the surface being inspected. The thermal response of the surface is measured with an infrared imaging system, which captures the emission from the surface and converts it to a color-coded map of the surface temperature distribution. Consider the case where a uniform flux of 5 kW/m^2 is applied to the top skin of an airplane wing initially at 20°C. The opposite side of the 15-mm-thick skin is adjacent to stagnant air and can be treated as well insulated. The density and specific heat of the skin material are 1200 kg/m^3 and 1200 J/kg · K, respectively. The effective thermal conductivity of the intact skin material is k_1 = 1.6 W/m · K. Contact resistances develop internal to the structure as a result of delamination between the fiber mats and the bonding material, leading to a reduced effective thermal conductivity of $k_2 = 1.1$ W/m · K. Determine the surface temperature of the component after 10 and 100 seconds of irradiation for (a) an area where the material is structurally intact and (b) an adjacent area where delamination has occurred within the wing.



5.89 Problem 4.9 addressed radioactive wastes stored underground in a spherical container. Because of uncertainty in the thermal properties of the soil, it is desired to measure the steady-state temperature using a test container (identical to the real container) that is equipped with internal electrical heaters. Estimate how long it will take the test container to come within 10°C of its steady-state value, assuming it is buried very far underground. Use the soil properties from Table A.3 in your analysis.

5.90 A thermal storage medium consists of a large mass of limestone. Ambient air is pumped through a 30-mm-high, 5-m-long, 5-m-wide fissure within the limestone at a velocity of 2 m/s, providing a heat transfer coefficient between the air and the limestone of $h = 10$ W/m^2 · K. Over a 24-hour time period, the temperature of the surface of the limestone adjacent to the air stream near the inlet exhibits periodic behavior with maximum and minimum values of 302 and 298 K, respectively.

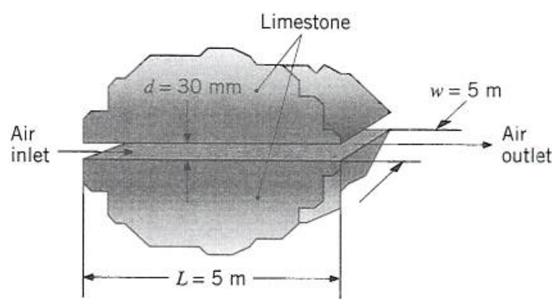
(a) Determine the minimum and maximum values of the temperature of the air near the inlet.

(b) Determine the maximum heat flux to the air during a 24-hour time period near the inlet. Determine the air inlet and outlet temperatures at the time corresponding to the maximum air heat flux.

(c) Determine the air outlet temperatures at the times corresponding to the maximum and minimum air inlet temperatures.

(d) Plot the inlet air temperature, the limestone surface temperature, and the heat transfer rate to the limestone over 48 hours.

(e) What is the required thickness of limestone necessary to ensure that the limestone may be viewed as a semi-infinite medium?

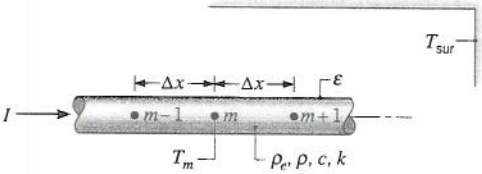


5.91 Consider the experimental measurement of Example 5.8. It is desired to measure the thermal conductivity of an extremely thin sample of the same nanostructured material having the same length and width. To minimize experimental uncertainty, the experimenter wishes to keep the amplitude of the temperature response, ΔT, above a value of 0.1°C. What is the minimum sample thickness that can be measured? Assume the properties of the thin sample and the magnitude of the applied heating rate are the same as those measured and used in Example 5.8.

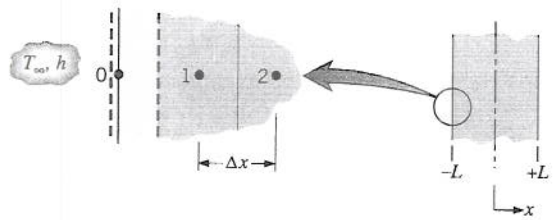
Finite-Difference Equations: Derivations

5.92 The stability criterion for the explicit method requires that the coefficient of the T_m^p term of the one-dimensional, finite-difference equation be zero or positive. Consider the situation for which the temperatures at the two neighboring nodes (T_{m-1}^p, T_{m+1}^p) are 100°C while the center node (T_m^p) is at 50°C. Show that for values of $Fo > \frac{1}{2}$ the finite-difference equation will predict a value of T_m^{p+1} that violates the second law of thermodynamics.

5.93 A thin rod of diameter D is initially in equilibrium with its surroundings, a large vacuum enclosure at temperature T_{sur}. Suddenly an electrical current I (A) is passed through the rod having an electrical resistivity ρ_e and emissivity ε. Other pertinent thermophysical properties are identified in the sketch. Derive the transient, finite-difference equation for node m.

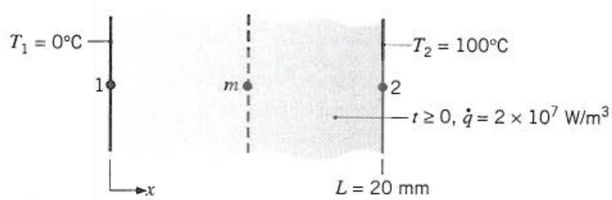


5.94 A one-dimensional slab of thickness $2L$ is initially at a uniform temperature T_i. Suddenly, electric current is passed through the slab causing a uniform volumetric heating $\dot{q}$ (W/m^3). At the same time, both outer surfaces ($x = \pm L$) are subjected to a convection process at T_∞ with a heat transfer coefficient h.



Write the finite-difference equation expressing conservation of energy for node 0 located on the outer surface at $x = -L$. Rearrange your equation and identify any important dimensionless coefficients.

5.95 A plane wall ($\rho = 4000$ kg/m^3, $c_p = 500$ J/kg · K, $k = 10$ W/m · K) of thickness $L = 20$ mm initially has a linear, steady-state temperature distribution with boundaries maintained at $T_1 = 0$°C and $T_2 = 100$°C. Suddenly, an electric current is passed through the wall, causing uniform energy generation at a rate $\dot{q} = 2 \times 10^7$ W/m^3. The boundary conditions T_1 and T_2 remain fixed.



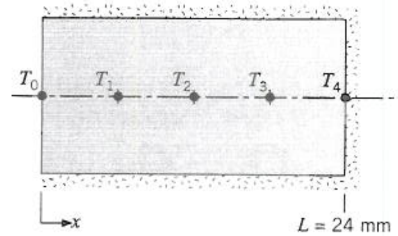
(a) On T–x coordinates, sketch temperature distributions for the following cases: (i) initial condition ($t \leq 0$); (ii) steady-state conditions ($t \to \infty$), assuming that the maximum temperature in the wall exceeds T_2; and (iii) for two intermediate times. Label all important features of the distributions.

(b) For the system of three nodal points shown schematically (1, m, 2), define an appropriate control volume for node m and, identifying all relevant processes, derive the corresponding finite-difference equation using either the *explicit* or *implicit* method.

(c) With a time increment of $\Delta t = 5$ s, use the finite-difference method to obtain values of T_m for the first 45 s of elapsed time. Determine the corresponding heat fluxes at the boundaries, that is, q''_x (0, 45 s) and q''_x (20 mm, 45 s).

(d) To determine the effect of mesh size, repeat your analysis using grids of 5 and 11 nodal points ($\Delta x = 5.0$ and 2.0 mm, respectively).

5.96 A round solid cylinder made of a plastic material ($\alpha = 6 \times 10^{-7}$ m^2/s) is initially at a uniform temperature of 20°C and is well insulated along its lateral surface and at one end. At time $t = 0$, heat is applied to the left boundary causing T_0 to increase linearly with time at a rate of 1°C/s.

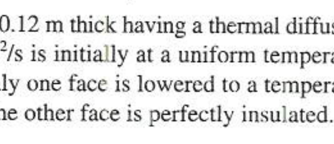


(a) Using the explicit method with $Fo = \frac{1}{2}$, derive the finite-difference equations for nodes 1, 2, 3, and 4.

(b) Format a table with headings of p, t (s), and the nodal temperatures T_0 to T_4. Determine the surface temperature T_0 when $T_4 = 35$°C.

5.97 Derive the explicit finite-difference equation for an interior node for three-dimensional transient conduction. Also determine the stability criterion. Assume constant properties and equal grid spacing in all three directions.

5.98 Derive the transient, two-dimensional finite-difference equation for the temperature at nodal point 0 located on the boundary between two different materials.



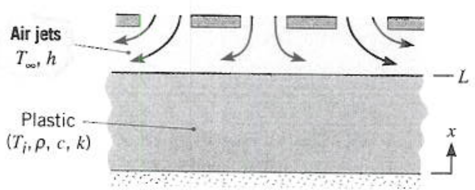
Finite-Difference Solutions: One-Dimensional Systems

5.99 A wall 0.12 m thick having a thermal diffusivity of 1.5×10^{-6} m^2/s is initially at a uniform temperature of 85°C. Suddenly one face is lowered to a temperature of 20°C, while the other face is perfectly insulated.

(a) Using the explicit finite-difference technique with space and time increments of 30 mm and 300 s, respectively, determine the temperature distribution at $t = 45$ min.

(b) With $\Delta x = 30$ mm and $\Delta t = 300$ s, compute $T(x, t)$ for $0 \le t \le t_{ss}$, where t_{ss} is the time required for the temperature at each nodal point to reach a value that is within 1°C of the steady-state temperature. Repeat the foregoing calculations for $\Delta t = 75$ s. For each value of Δt, plot temperature histories for each face and the midplane.

5.100 A molded plastic product ($\rho = 1200\ \text{kg/m}^3$, $c = 1500\ \text{J/kg} \cdot \text{K}$, $k = 0.30\ \text{W/m} \cdot \text{K}$) is cooled by exposing one surface to an array of air jets, while the opposite surface is well insulated. The product may be approximated as a slab of thickness $L = 60$ mm, which is initially at a uniform temperature of $T_i = 80$°C. The air jets are at a temperature of $T_\infty = 20$°C and provide a uniform convection coefficient of $h = 100\ \text{W/m}^2 \cdot \text{K}$ at the cooled surface.



Using a finite-difference solution with a space increment of $\Delta x = 6$ mm, determine temperatures at the cooled and insulated surfaces after 1 hour of exposure to the gas jets.

5.101 The plane wall of Problem 2.48 ($k = 50\ \text{W/m} \cdot \text{K}$, $\alpha = 1.5 \times 10^{-6}\ \text{m}^2/\text{s}$) has a thickness of $L = 40$ mm and an initial uniform temperature of $T_o = 25$°C. Suddenly, the boundary at $x = L$ experiences heating by a fluid for which $T_\infty = 50$°C and $h = 1000\ \text{W/m}^2 \cdot \text{K}$, while heat is uniformly generated within the wall at $\dot{q} = 1 \times 10^7\ \text{W/m}^3$. The boundary at $x = 0$ remains at T_o.

(a) With $\Delta x = 4$ mm and $\Delta t = 1$ s, plot temperature distributions in the wall for (i) the initial condition, (ii) the steady-state condition, and (iii) two intermediate times.

(b) On q_x''–t coordinates, plot the heat flux at $x = 0$ and $x = L$. At what elapsed time is there zero heat flux at $x = L$?

5.102 Consider the fuel element of Example 5.9. Initially, the element is at a uniform temperature of 250°C with no heat generation. Suddenly, the element is inserted into the reactor core causing a uniform volumetric heat generation rate of $\dot{q} = 10^8\ \text{W/m}^3$. The surfaces are convectively cooled with $T_\infty = 250$°C and $h = 1100\ \text{W/m}^2 \cdot \text{K}$. Using the explicit method with a space increment of 2 mm, determine the temperature distribution 1.5 s after the element is inserted into the core.

5.103 Consider the fuel element of Example 5.9, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7\ \text{W/m}^3$, until the generation rate suddenly changes to $\dot{q}_2 = 2 \times 10^7\ \text{W/m}^3$. Use the *Finite-Difference Equations, One-Dimensional, Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the 6 nodes, with $\Delta x = 2$ mm, as shown in the example.

(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example.

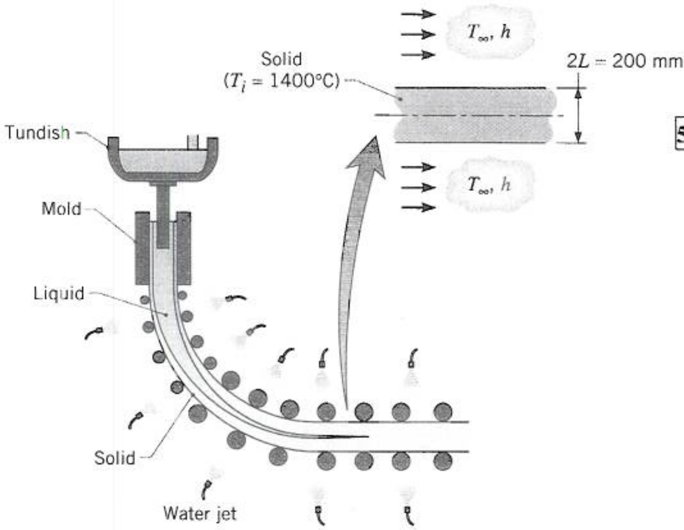
(b) Use the *Explore* and *Graph* options of *IHT* to calculate and plot temperature histories at the midplane (00) and surface (05) nodes for $0 \le t \le 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

5.104 Consider the fuel element of Example 5.9, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7\ \text{W/m}^3$, until the generation rate suddenly changes to $\dot{q}_2 = 2 \times 10^7\ \text{W/m}^3$. Use the finite-element software *FEHT* to obtain the following solutions.

(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example. *Hint*: First determine the steady-state temperature distribution for $\dot{q}_1$, which represents the initial condition for the transient temperature distribution after the step change in power to $\dot{q}_2$. Next, in the *Setup* menu, click on *Transient*: in the *Specify/Internal Generation* box, change the value to $\dot{q}_2$; and in the *Run* command, click on *Continue* (not *Calculate*). See the *Run* menu in the *FEHT* Help section for background information on the *Continue* option.

(b) Use your *FEHT* model to plot temperature histories at the midplane and surface for $0 \le t \le 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

5.105 In a thin-slab, continuous casting process, molten steel leaves a mold with a thin solid shell, and the molten material solidifies as the slab is quenched by water jets en route to a section of rollers. Once fully solidified, the slab continues to cool as it is brought to an acceptable handling temperature. It is this portion of the process that is of interest.



Consider a 200-mm-thick solid slab of steel (ρ = 7800 kg/m^3, c = 700 J/kg · K, k = 30 W/m · K), initially at a uniform temperature of T_i = 1400°C. The slab is cooled at its top and bottom surfaces by water jets (T_∞ = 50°C), which maintain an approximately uniform convection coefficient of h = 5000 W/m^2 · K at both surfaces. Using a finite-difference solution with a space increment of Δx = 1 mm, determine the time required to cool the surface of the slab to 200°C. What is the corresponding temperature at the midplane of the slab? If the slab moves at a speed of V = 15 mm/s, what is the required length of the cooling section?

5.106 A very thick plate with thermal diffusivity 5.6 × 10^{-6} m^2/s and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is 100 W/m^2 · K. Using the finite-difference method with a space increment of Δx = 15 mm and a time increment of 18 s, determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

5.107 Referring to Example 5.10, Comment 4, consider a sudden exposure of the surface to large surroundings at an elevated temperature (T_{sur}) and to convection (T_∞, h).

(a) Derive the explicit, finite-difference equation for the surface node in terms of *Fo*, *Bi*, and Bi_r.

(b) Obtain the stability criterion for the surface node. Does this criterion change with time? Is the criterion more restrictive than that for an interior node?

(c) A thick slab of material (k = 1.5 W/m · K, α = 7 × 10^{-7} m^2/s, ε = 0.9), initially at a uniform temperature of 27°C, is suddenly exposed to large surroundings at 1000 K. Neglecting convection and using a space increment of 10 mm, determine temperatures at the surface and 30 mm from the surface after an elapsed time of 1 min.

5.108 Consider the thick slab of copper in Example 5.10, which is initially at a uniform temperature of 20°C and is suddenly exposed to a net radiant flux of 3 × 10^5 W/m^2. Use the *Finite-Difference Equations/One-Dimensional/Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the interior nodes. In your analysis, use a space increment of Δx = 37.5 mm with a total of 17 nodes (00–16), and a time increment of Δt = 1.2 s. For the surface node 00, use the finite-difference equation derived in Section 2 of the Example.

(a) Calculate the 00 and 04 nodal temperatures at t = 120 s, that is, T(0, 120 s) and T(0.15 m, 120 s), and compare the results with those given in Comment 1 for the exact solution. Will a time increment of 0.12 s provide more accurate results?

(b) Plot temperature histories for x = 0, 150, and 600 mm, and explain key features of your results.

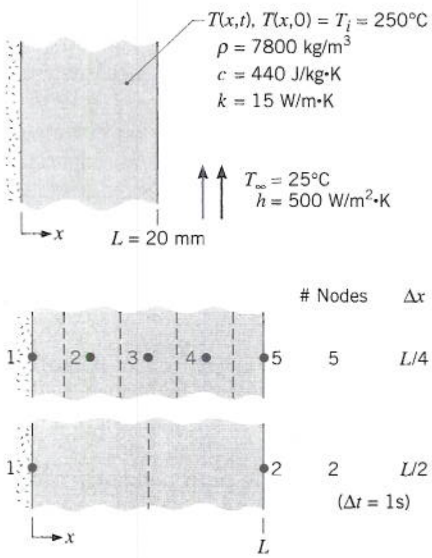
5.109 Consider the thick slab of copper in Example 5.10, which is initially at a uniform temperature of 20°C and is suddenly exposed to large surroundings at 1000°C (instead of a prescribed heat flux).

(a) For a surface emissivity of 0.94, calculate the temperatures T(0, 120 s) and T(0.15 m, 120 s) using the finite-element software *FEHT*. *Hint*: In the *Convection Coefficient* box of the *Specify/Boundary Conditions* menu of *FEHT*, enter the linearized radiation coefficient (see Equation 1.9) for the surface (x = 0). Enter the temperature of the surroundings in the *Fluid Temperature* box. See also the *Help* section on *Entering Equations*. Click on *Setup/Temperatures in K* to enter all temperatures in kelvins.

(b) Plot the temperature histories for x = 0, 150, and 600 mm, and explain key features of your results.

5.110 Consider the composite wall of Problem 2.53. In part (d), you are asked to sketch the temperature histories at x = 0, L during the transient period between cases 2 and 3. Calculate and plot these histories using the finite-element method of *FEHT*, the finite-difference method of *IHT* (with Δx = 5 mm and Δt = 1.2 s), and/or an alternative procedure of your choice. If you use more than one method, compare the respective results. Note that, in using *FEHT* or *IHT*, a look-up table must be created for prescribing the variation of the heater flux with time (see the appropriate *Help* section for guidance).

5.111 In Section 5.5, the one-term approximation to the series solution for the temperature distribution was developed for a plane wall of thickness $2L$ that is initially at a uniform temperature and suddenly subjected to convection heat transfer. If $Bi < 0.1$, the wall can be approximated as isothermal and represented as a lumped capacitance (Equation 5.7). For the conditions shown schematically, we wish to compare predictions based on the one-term approximation, the lumped capacitance method, and a finite-difference solution.

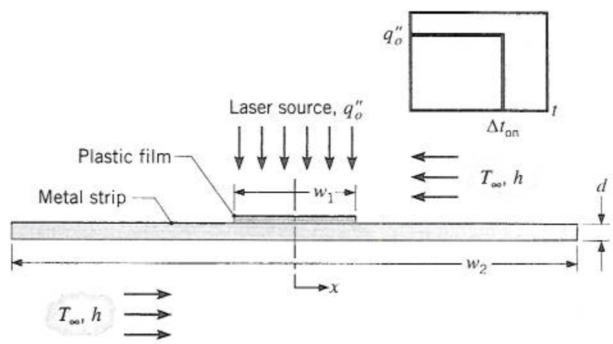


(a) Determine the midplane, $T(0, t)$, and surface, $T(L, t)$, temperatures at $t = 100$, 200, and 500 s using the one-term approximation to the series solution, Equation 5.40. What is the Biot number for the system?

(b) Treating the wall as a lumped capacitance, calculate the temperatures at $t = 50$, 100, 200, and 500 s. Did you expect these results to compare favorably with those from part (a)? Why are the temperatures considerably higher?

(c) Consider the 2- and 5-node networks shown schematically. Write the implicit form of the finite-difference equations for each network, and determine the temperature distributions for $t = 50$, 100, 200, and 500 s using a time increment of $\Delta t = 1$ s. You may use *IHT* to solve the finite-difference equations by representing the rate of change of the nodal temperatures by the intrinsic function, Der(T, t). Prepare a table summarizing the results of parts (a), (b), and (c). Comment on the relative differences of the predicted temperatures. *Hint*: See the *Solver/Intrinsic Functions* section of *IHT/Help* or the *IHT Examples* menu (Example 5.2) for guidance on using the Der(T, t) function.

5.112 Consider the bonding operation described in Problem 3.103, which was analyzed under steady-state conditions. In this case, however, the laser will be used to heat the film for a prescribed period of time, creating the transient heating situation shown in the sketch.



The strip is initially at 25°C and the laser provides a uniform flux of 85,000 W/m^2 over a time interval of $\Delta t_{on} = 10$ s. The system dimensions and thermophysical properties remain the same, but the convection coefficient to the ambient air at 25°C is now 100 W/m$^2\cdot$K and $w_1 = 44$ mm.

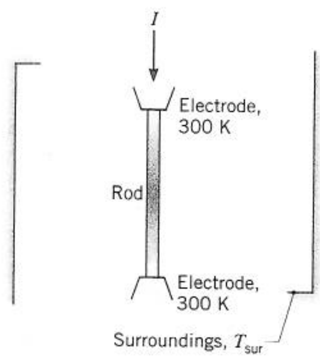
Using an implicit finite-difference method with $\Delta x = 4$ mm and $\Delta t = 1$ s, obtain temperature histories for $0 \leq t \leq 30$ s at the center and film edge, $T(0, t)$ and $T(w_1/2, t)$, respectively, to determine if the adhesive is satisfactorily cured above 90°C for 10 s and if its degradation temperature of 200°C is exceeded.

5.113 One end of a stainless steel (AISI 316) rod of diameter 10 mm and length 0.16 m is inserted into a fixture maintained at 200°C. The rod, covered with an insulating sleeve, reaches a uniform temperature throughout its length. When the sleeve is removed, the rod is subjected to ambient air at 25°C such that the convection heat transfer coefficient is 30 W/m$^2\cdot$K.

(a) Using the explicit finite-difference technique with a space increment of $\Delta x = 0.016$ m, estimate the time required for the midlength of the rod to reach 100°C.

(b) With $\Delta x = 0.016$ m and $\Delta t = 10$ s, compute $T(x, t)$ for $0 \leq t \leq t_1$, where t_1 is the time required for the midlength of the rod to reach 50°C. Plot the temperature distribution for $t = 0$, 200 s, 400 s, and t_1.

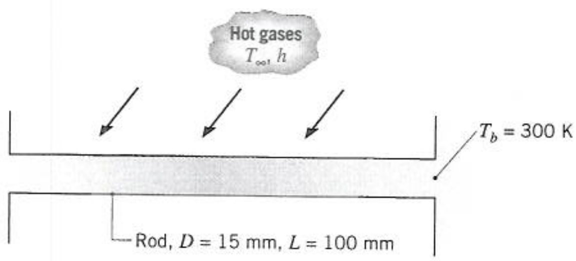
5.114 A tantalum rod of diameter 3 mm and length 120 mm is supported by two electrodes within a large vacuum enclosure. Initially the rod is in equilibrium with the electrodes and its surroundings, which are maintained at 300 K. Suddenly, an electrical current, $I = 80$ A, is passed through the rod. Assume the emissivity of the rod is 0.1 and the electrical resistivity is $95 \times 10^{-8}\ \Omega \cdot \text{m}$. Use Table A.1 to obtain the other thermophysical properties required in your solution. Use a finite-difference method with a space increment of 10 mm.



(a) Estimate the time required for the midlength of the rod to reach 1000 K.

(b) Determine the steady-state temperature distribution and estimate approximately how long it will take to reach this condition.

5.115 A support rod ($k = 15$ W/m · K, $\alpha = 4.0 \times 10^{-6}$ m²/s) of diameter $D = 15$ mm and length $L = 100$ mm spans a channel whose walls are maintained at a temperature of $T_b = 300$ K. Suddenly, the rod is exposed to a cross flow of hot gases for which $T_\infty = 600$ K and $h = 75$ W/m² · K. The channel walls are cooled and remain at 300 K.



(a) Using an appropriate numerical technique, determine the thermal response of the rod to the convective heating. Plot the midspan temperature as a function of elapsed time. Using an appropriate analytical model of the rod, determine the steady-state temperature distribution and compare the result with that obtained numerically for very long elapsed times.

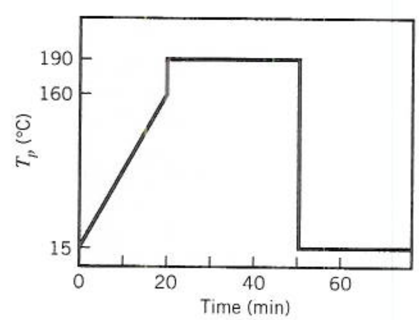
(b) After the rod has reached steady-state conditions, the flow of hot gases is suddenly terminated, and the rod cools by free convection to ambient air at $T_\infty = 300$ K and by radiation exchange with large surroundings at $T_{sur} = 300$ K. The free convection coefficient can be expressed as h (W/m² · K) $= C\ \Delta T^n$, where $C = 4.4$ W/m² · K$^{1.188}$ and $n = 0.188$. The emissivity of the rod is 0.5. Determine the subsequent thermal response of the rod. Plot the midspan temperature as a function of cooling time, and determine the time required for the rod to reach a *safe-to-touch* temperature of 315 K.

5.116 Consider the acceleration-grid foil ($k = 40$ W/m · K, $\alpha = 3 \times 10^{-5}$ m²/s, $\varepsilon = 0.45$) of Problem 4.72. Develop an implicit, finite-difference model of the foil, which can be used for the following purposes.

(a) Assuming the foil to be at a uniform temperature of 300 K when the ion beam source is activated, obtain a plot of the midspan temperature–time history. At what elapsed time does this point on the foil reach a temperature within 1 K of the steady-state value?

(b) The foil is operating under steady-state conditions when, suddenly, the ion beam is deactivated. Obtain a plot of the subsequent midspan temperature–time history. How long does it take for the hottest point on the foil to cool to 315 K, a *safe-to-touch* condition?

5.117 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.45 and described further in Problem 5.46. A finite-difference method of solution is sought with two additional considerations. First, the book is to be treated as having distributed, rather than lumped, characteristics, by using a grid spacing of $\Delta x = 2.36$ mm with nodes at the center of the individual circuit board or plate. Second, rather than bringing the platens to 190°C in one sudden change, the heating schedule $T_p(t)$ shown below is to be used in order to minimize excessive thermal stresses induced by rapidly changing thermal gradients in the vicinity of the platens.

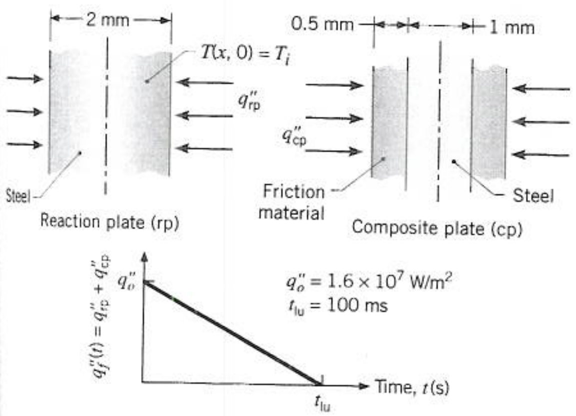


(a) Using a time increment of $\Delta t = 60$ s and the implicit method, find the temperature history of the midplane of the book and determine whether curing will occur (170°C for 5 min).

(b) Following the reduction of the platen temperatures to 15°C ($t = 50$ min), how long will it take for the midplane of the book to reach 37°C, a safe temperature at which the operator can begin unloading the press?

(c) Validate your program code by using the heating schedule of a sudden change of platen temperature from 15 to 190°C and compare results with those from an appropriate analytical solution (see Problem 5.46).

5.118 Common transmission failures result from the glazing of clutch surfaces by deposition of oil oxidation and decomposition products. Both the oxidation and decomposition processes depend on temperature histories of the surfaces. Because it is difficult to measure these surface temperatures during operation, it is useful to develop models to predict clutch-interface thermal behavior. The relative velocity between mating clutch plates, from the initial engagement to the zero-sliding (*lock-up*) condition, generates heat that is transferred to the plates. The relative velocity decreases at a constant rate during this period, producing a heat flux that is initially very large and decreases linearly with time, until lock-up occurs. Accordingly, $q''_f = q''_o[1 - (t/t_{lu})]$, where $q''_o = 1.6 \times 10^7$ W/m^2 and $t_{lu} = 100$ ms is the lock-up time. The plates have an initial uniform temperature of $T_i = 40$°C, when the prescribed frictional heat flux is suddenly applied to the surfaces. The reaction plate is fabricated from steel, while the composite plate has a thinner steel center section bonded to low-conductivity friction material layers. The thermophysical properties are $\rho_s = 7800$ kg/m^3, $c_s = 500$ J/kg · K, and $k_s = 40$ W/m · K for the steel and $\rho_{fm} = 1150$ kg/m^3, $c_{fm} = 1650$ J/kg · K, and $k_{fm} = 4$ W/m · K for the friction material.



(a) On $T - t$ coordinates, sketch the temperature history at the midplane of the reaction plate, at the interface between the clutch pair, and at the midplane of the composite plate. Identify key features.

(b) Perform an energy balance on the clutch pair over the time interval $\Delta t = t_{lu}$ to determine the steady-state temperature resulting from clutch engagement. Assume negligible heat transfer from the plates to the surroundings.

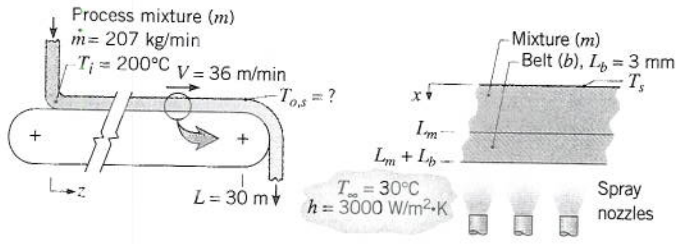
[(c)] Compute and plot the three temperature histories of interest using the finite-element method of *FEHT* or the finite-difference method of *IHT* (with $\Delta x = 0.1$ mm and $\Delta t = 1$ ms). Calculate and plot the frictional heat fluxes to the reaction and composite plates, q''_{rp} and q''_{cp}, respectively, as a function of time. Comment on features of the temperature and heat flux histories. Validate your model by comparing predictions with the results from part (b). *Note*: Use of both *FEHT* and *IHT* requires creation of a look-up data table for prescribing the heat flux as a function of time.

[5.119] Heat transfer is not an intuitive process muses the Curious Cook. Does doubling the thickness of a hamburger approximately double the cooking time? What effect does the initial temperature have on cooking time? To answer these questions, develop a model to do virtual cooking of meat of thickness $2L$ in a double-sided grill. The meat is initially at 20°C when it is placed in the grill and both sides experience convection heat transfer characterized by an ambient temperature of 100°C and a convection coefficient of 5000 W/m^2 · K. Assume the meat to have the properties of liquid water at 300 K and to be properly cooked when the center temperature is 60°C.

(a) For hamburgers of thickness $2L$ = 10, 20, and 30 mm, calculate the time for the center to reach the required cooking temperature of 60°C. Determine a relationship between the cooking time and the thickness. For your solution, use the finite-element method of *FEHT*, the ready-to-solve model in the *Models/Transient Conduction/Plane Wall* section of *IHT*, or a numerical procedure of your choice. For one of the thicknesses, use an appropriate analytical solution to validate your numerical results.

(b) Without performing a detailed numerical solution, but drawing on the results of part (a), what can you say about the effect on the cooking time of changing the initial temperature of the meat from 20°C to 5°C? You may use your numerical model from part (a) to confirm your assessment.

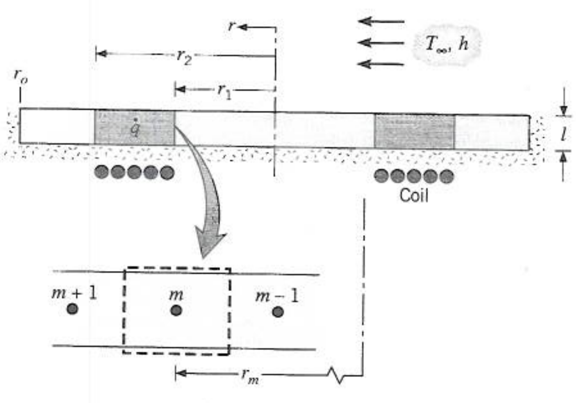
5.120 A process mixture at 200°C flows at a rate of 207 kg/min onto a conveyor belt of 3-mm thickness, 1-m width, and 30-m length traveling with a velocity of 36 m/min. The underside of the belt is cooled by a water spray at a temperature of 30°C, and the convection coefficient is 3000 W/m² · K. The thermophysical properties of the process mixture are $\rho_m = 960$ kg/m³, $c_m = 1700$ J/kg · K, and $k_m = 1.5$ W/m · K, while the properties for the conveyor (metallic) belt are $\rho_b = 8000$ kg/m³, $c_b = 460$ J/kg · K, and $k_b = 15$ W/m · K.



Using the finite-difference method of *IHT* ($\Delta x = 0.5$ mm, $\Delta t = 0.05$ s), the finite-element method of *FEHT*, or a numerical procedure of your choice, calculate the surface temperature of the mixture at the end of the conveyor belt, $T_{o,s}$. Assume negligible heat transfer to the ambient air by convection or by radiation to the surroundings.

Finite-Difference Equations: Cylindrical Coordinates

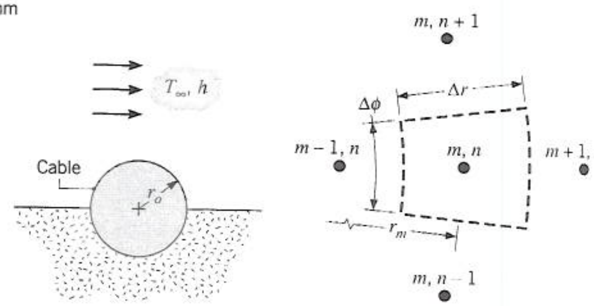
5.121 A thin circular disk is subjected to induction heating from a coil, the effect of which is to provide a uniform heat generation within a ring section as shown. Convection occurs at the upper surface, while the lower surface is well insulated.



(a) Derive the transient, finite-difference equation for node m, which is within the region subjected to induction heating.

(b) On T–r coordinates sketch, in a qualitative manner, the steady-state temperature distribution, identifying important features.

5.122 An electrical cable, experiencing a uniform volumetric generation $\dot{q}$, is half buried in an insulating material while the upper surface is exposed to a convection process (T_∞, h).



(a) Derive the explicit, finite-difference equations for an interior node (m, n), the center node $(m = 0)$, and the outer surface nodes (M, n) for the convection and insulated boundaries.

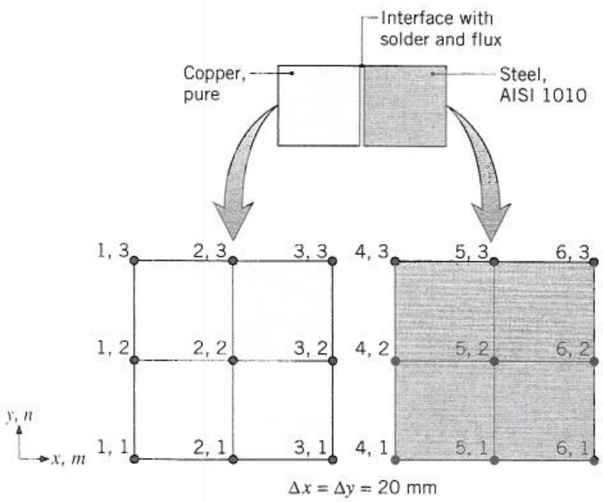
(b) Obtain the stability criterion for each of the finite-difference equations. Identify the most restrictive criterion.

Finite-Difference Solutions: Two-Dimensional Systems

5.123 Two very long (in the direction normal to the page) bars having the prescribed initial temperature distributions are to be soldered together. At time $t = 0$, the $m = 3$ face of the copper (pure) bar contacts the $m = 4$ face of the steel (AISI 1010) bar. The solder and flux act as an interfacial layer of negligible thickness and effective contact resistance $R''_{t,c} = 2 \times 10^{-5}$ m² · K/W.

Initial Temperatures (K)

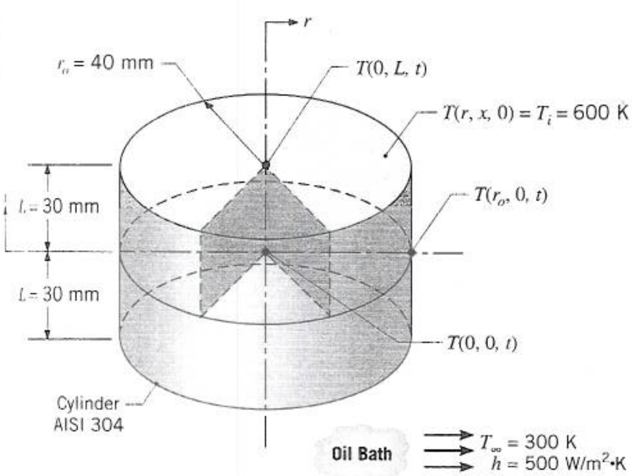
n/m	1	2	3	4	5	6
1	700	700	700	1000	900	800
2	700	700	700	1000	900	800
3	700	700	700	1000	900	800



(a) Derive the explicit, finite-difference equation in terms of Fo and $Bi_c = \Delta x/kR''_{t,c}$ for $T_{4,2}$ and determine the corresponding stability criterion.

(b) Using $Fo = 0.01$, determine $T_{4,2}$ one time step after contact is made. What is Δt? Is the stability criterion satisfied?

5.124 In a manufacturing process, stainless steel cylinders (AISI 304) initially at 600 K are quenched by submersion in an oil bath maintained at 300 K with $h = 500$ $W/m^2 \cdot K$. Each cylinder is of length $2L = 60$ mm and diameter $D = 80$ mm. Use the *ready-to-solve* model in the *Examples* menu of *FEHT* to obtain the following solutions.



(a) Calculate the temperatures, $T(r, x, t)$, after 3 min at the cylinder center, $T(0, 0, 3 \text{ min})$, at the center of a circular face, $T(0, L, 3 \text{ min})$, and the midheight of the side, $T(r_o, 0, 3 \text{ min})$.

(b) Plot the temperature history at the center, $T(0, 0, t)$, and at the midheight of the side, $T(r_o, 0, t)$, for $0 \le t \le 10$ min using the *View/Temperatures vs. Time* command. Comment on the gradients occurring at these locations and what effect they might have on phase transformations and thermal stresses.

(c) Having solved the model for a total integration time of 10 min in part (b), now use the *View/Temperature Contours* command with the shaded band option for the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the cylinder cools during the quench process. Describe the major features of the cooling process revealed by this display. Use other options of this command to create a 10-isotherm temperature distribution for $t = 3$ min.

(d) For the location of part (a), calculate the temperatures after 3 min if the convection coefficient is doubled ($h = 1000$ $W/m^2 \cdot K$). Also, for convection coefficients of 500 and 1000 $W/m^2 \cdot K$, determine how long the cylinder needs to remain in the oil bath to achieve a *safe-to-touch* surface temperature of 316 K. Tabulate and comment on the results of parts (a) and (d).

5.125 Consider the system of Problem 4.55. Initially with no flue gases flowing, the walls ($\alpha = 5.5 \times 10^{-7}$ m^2/s) are at a uniform temperature of 25°C. Using the implicit, finite-difference method with a time increment of 1 h, find the temperature distribution in the wall 5, 10, 50, and 100 h after introduction of the flue gases.

5.126 Consider the system of Problem 4.73. Initially, the ceramic plate ($\alpha = 1.5 \times 10^{-6}$ m^2/s) is at a uniform temperature of 30°C, and suddenly the electrical heating elements are energized. Using the implicit, finite-difference method, estimate the time required for the difference between the surface and initial temperatures to reach 95% of the difference for steady-state conditions. Use a time increment of 2 s.

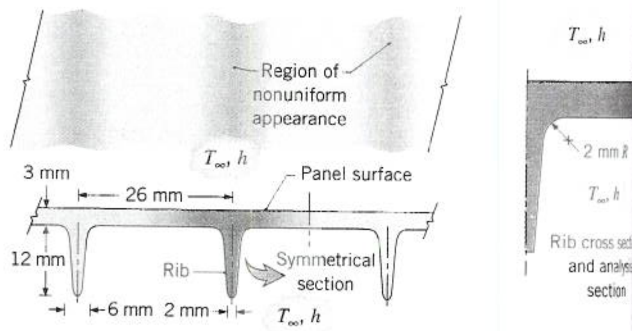
5.127 Consider the thermal conduction module and operating conditions of Problem 4.76. To evaluate the transient response of the cold plate, which has a thermal diffusivity of $\alpha = 75 \times 10^{-6}$ m^2/s, assume that, when the module is activated at $t = 0$, the initial temperature of the cold plate is $T_i = 15$°C and a uniform heat flux of $q''_o = 10^5$ W/m^2 is applied at its base. Using the implicit finite-difference method and a time increment of $\Delta t = 0.1$ s, compute the designated nodal temperatures as a function of time. From the temperatures computed at a particular time, evaluate the ratio of the rate of heat transfer by convection to the water to the heat input at the base. Terminate the calculations when this ratio reaches 0.99. Print the temperature field at 5-s intervals and at the time for which the calculations are terminated.

5.128 The operations manager for a metals processing plant anticipates the need to repair a large furnace and has come to you for an estimate of the time required for the furnace interior to cool to a safe working temperature. The furnace is cubical with a 16-m interior dimension and 1-m thick walls for which $\rho = 2600$ kg/m^3, $c = 960$ J/kg · K, and $k = 1$ W/m · K. The operating temperature of the furnace is 900°C, and the outer surface experiences convection with ambient air at 25°C and a convection coefficient of 20 W/m^2 · K.

(a) Use a numerical procedure to estimate the time required for the inner surface of the furnace to cool to a safe working temperature of 35°C. *Hint*: Consider a two-dimensional cross-section of the furnace, and perform your analysis on the smallest symmetrical section.

(b) Anxious to reduce the furnace downtime, the operations manager also wants to know what effect circulating ambient air through the furnace would have on the cool-down period. Assume equivalent convection conditions for the inner and outer surfaces.

5.129 The door panel of an automobile is fabricated by a plastic hot-extrusion process resulting in the ribbed cross section shown schematically. Following a process involving air-cooling, painting, and baking, the panel is ready for assembly on the vehicle. However, upon visual inspection, the rib pattern is evident on the outer surface. In regions over the ribs, the paint has an "orange peel" appearance, making the door panel unacceptable for use. The apparent reason for this defect is the variable microfinish caused by differential cooling rates at the surface of the panel, which affect adherence of the paint. Your assignment is to estimate the panel surface temperature distribution as a function of cooling time.



Use the finite-element software *FEHT* to obtain the temperature distribution in the symmetrical section shown schematically. The panel is ejected from the extrusion press at a uniform temperature of 275°C and is allowed to cool on a transport table, where the air temperature is 25°C and the convection coefficient is 10 W/m^2 · K. The thermophysical properties of the extruded plastic are $\rho = 1050$ kg/m^3, $c = 800$ J/kg · K, and $k = 0.5$ W/m · K.

(a) Using the *View/Temperature vs. Time* command, plot temperature histories for selected locations on the panel surface. Is there noticeable differential cooling in the region above the rib? If so, what is the cause? *Hint*: When you draw the shape outline in *FEHT*, represent the fillet region by six or seven segments that approximate the fillet radius. Then draw element lines to form triangular elements.

(b) Use the *View/Temperature Contours* command with the shaded band option to plot the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the panel cools. Describe major features of the cooling process observed from the display. Use other options of this command to create a 10-isotherm temperature distribution for a time that illustrates some of the foregoing important features. How would you redesign the ribbed panel to reduce this thermally induced defect, while still retaining the stiffening function required of the ribs?