conditions are such that the use of a shape factor, dimensionless conduction heat rate, or an exact solution is not possible, and it is necessary to use a finite-difference or finite-element solution. You should appreciate the inherent nature of the *discretization process* and know how to formulate and solve the finite-difference equations for the discrete points of a nodal network. You may test your understanding of related concepts by addressing the following questions.

- What is an *isotherm*? What is a *heat flow line*? How are the two lines related geometrically?
- What is an *adiabat*? How is it related to a line of symmetry? How is it intersected by an isotherm?
- What parameters characterize the effect of geometry on the relationship between the heat rate and the overall temperature difference for steady conduction in a two-dimensional system? How are these parameters related to the conduction resistance?
- What is represented by the temperature of a *nodal point*, and how does the accuracy of a nodal temperature depend on prescription of the *nodal network*?

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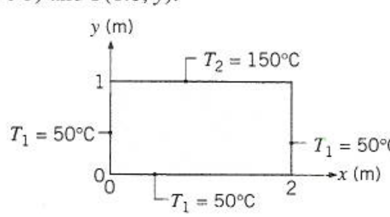
Problems

Exact Solutions

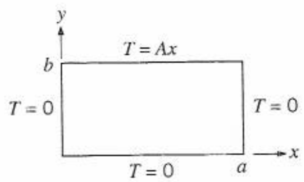
4.1 In the method of separation of variables (Section 4.2) for two-dimensional, steady-state conduction, the separation constant λ^2 in Equations 4.6 and 4.7 must be a positive constant. Show that a negative or zero value of λ^2 will result in solutions that cannot satisfy the prescribed boundary conditions.

4.2 A two-dimensional rectangular plate is subjected to prescribed boundary conditions. Using the results of the exact solution for the heat equation presented in Section 4.2, calculate the temperature at the midpoint (1, 0.5) by considering the first five nonzero terms of the infinite series that must be evaluated. Assess the error resulting from using only the first three terms of the infinite series. Plot the temperature distributions $T(x, 0.5)$ and $T(1.0, y)$.

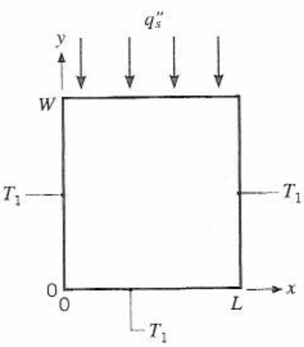


4.3 Consider the two-dimensional rectangular plate of Problem 4.2 having a thermal conductivity of 50 W/m · K. Beginning with the exact solution for the temperature distribution, derive an expression for the heat transfer rate per unit thickness from the plate along the lower surface ($0 \leq x \leq 2$, $y = 0$). Evaluate the heat rate considering the first five nonzero terms of the infinite series.

4.4 A two-dimensional rectangular plate is subjected to the boundary conditions shown. Derive an expression for the steady-state temperature distribution $T(x, y)$.



4.5 A two-dimensional rectangular plate is subjected to prescribed temperature boundary conditions on three sides and a uniform heat flux *into* the plate at the top surface. Using the general approach of Section 4.2, derive an expression for the temperature distribution in the plate.



Shape Factors and Dimensionless Conduction Heat Rates

4.6 Using the thermal resistance relations developed in Chapter 3, determine shape factor expressions for the following geometries:

(a) Plane wall, cylindrical shell, and spherical shell.

(b) Isothermal sphere of diameter D buried in an infinite medium.

4.7 Consider Problem 4.5 for the case where the plate is of square cross section, $W = L$.

(a) Derive an expression for the shape factor, S_{max}, associated with the *maximum* top surface temperature, such that $q = S_{max}\, k\, (T_{2,max} - T_1)$ where $T_{2,max}$ is the maximum temperature along $y = W$.

(b) Derive an expression for the shape factor, S_{avg}, associated with the *average* top surface temperature, $q = S_{avg} k(\overline{T}_2 - T_1)$ where $\overline{T}_2$ is the average temperature along $y = W$.

(c) Evaluate the shape factors that can be used to determine the maximum and average temperatures along $y = W$. Evaluate the maximum and average temperatures for $T_1 = 0°C$, $L = W = 10$ mm, $k = 20$ W/m · K, and $q''_s = 1000$ W/m^2.

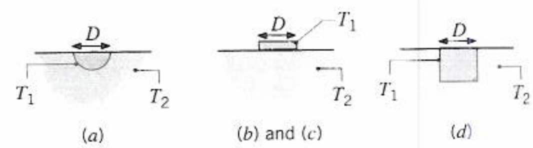
4.8 Based on the dimensionless conduction heat rates for cases 12–15 in Table 4.1*b*, find shape factors for the following objects having temperature T_1, located at the surface of a semi-infinite medium having temperature T_2. The surface of the semi-infinite medium is adiabatic.

(a) A buried hemisphere, flush with the surface.

(b) A disk on the surface. Compare your result to Table 4.1*a*, case 10.

(c) A square on the surface.

(d) A buried cube, flush with the surface.



4.9 Radioactive wastes are temporarily stored in a spherical container, the center of which is buried a distance of 10 m below the earth's surface. The outside diameter of the container is 2 m, and 500 W of heat are released as a result of the radioactive decay process. If the soil surface temperature is 20°C, what is the outside surface temperature of the container under steady-state conditions? On a sketch of the soil–container system drawn to scale, show representative isotherms and heat flow lines in the soil.

4.10 A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 1.5 m below the surface. The pipe has an outer diameter of 0.5 m and is insulated with a layer of cellular glass 100 mm thick. What is the heat loss per unit length of pipe under conditions for which heated oil at 120°C flows through the pipe and the surface of the earth is at a temperature of 0°C?

4.11 A long power transmission cable is buried at a depth (ground to cable centerline distance) of 2 m. The cable is encased in a thin-walled pipe of 0.1-m diameter, and to render the cable *superconducting* (essentially zero power dissipation), the space between the cable and pipe is filled with liquid nitrogen at 77 K. If the pipe is covered with a superinsulator ($k_i = 0.005$ W/m · K) of 0.05-m thickness and the surface of the earth ($k_g = 1.2$ W/m · K) is at 300 K, what is the cooling load in W/m that must be maintained by a cryogenic refrigerator per unit pipe length?

4.12 An electrical heater 100 mm long and 5 mm in diameter is inserted into a hole drilled normal to the surface of a large block of material having a thermal conductivity of 5 W/m · K. Estimate the temperature reached by the heater when dissipating 50 W with the surface of the block at a temperature of 25°C.

4.13 Two parallel pipelines spaced 0.5 m apart are buried in soil having a thermal conductivity of 0.5 W/m · K. The pipes have outer diameters of 100 and 75 mm with surface temperatures of 175°C and 5°C, respectively. Estimate the heat transfer rate per unit length between the two pipelines.

4.14 A tube of diameter 50 mm having a surface temperature of 85°C is embedded in the center plane of a concrete slab 0.1 m thick with upper and lower surfaces at 20°C. Using the appropriate tabulated relation for this configuration, find the shape factor. Determine the heat transfer rate per unit lengh of the tube.

4.15 Pressurized steam at 450 K flows through a long, thin-walled pipe of 0.5-m diameter. The pipe is enclosed in a concrete casing that is of square cross section and 1.5 m on a side. The axis of the pipe is centered in the casing, and the outer surfaces of the casing are maintained at 300 K. What is the heat loss per unit length of pipe?

4.16 Hot water at 85°C flows through a thin-walled copper tube of 30 mm diameter. The tube is enclosed by an eccentric cylindrical shell that is maintained at 35°C and has a diameter of 120 mm. The eccentricity, defined as the separation between the centers of the tube and shell, is 20 mm. The space between the tube and shell is filled with an insulating material having a thermal conductivity of 0.05 W/m · K. Calculate the heat loss per unit length of the tube and compare the result with the heat loss for a concentric arrangement.

4.17 A furnace of cubical shape, with external dimensions of 0.35 m, is constructed from a refractory brick (fireclay). If the wall thickness is 50 mm, the inner surface temperature is 600°C, and the outer surface temperature is 75°C, calculate the heat loss from the furnace.

4.18 The temperature distribution in laser-irradiated materials is determined by the power, size, and shape of the laser beam, along with the properties of the material being irradiated. The beam shape is typically Gaussian, and the local beam irradiation flux (often referred to as the laser *fluence*) is

$$q''(x, y) = q''(x = y = 0)\exp(-x/r_b)^2 \exp(-y/r_b)^2$$

The x and y coordinates determine the location of interest on the surface of the irradiated material. Consider the case where the center of the beam is located at $x = y = r = 0$. The beam is characterized by a radius, r_b, defined as the radial location where the local fluence is $q''(r_b) = q''(r = 0)/e \approx 0.368q''(r = 0)$.

A shape factor for Gaussian heating is $S = 2\pi^{1/2}r_b$, where S is defined in terms of $T_{1,\max} - T_2$ [Nissin, Y. I., A. Lietoila, R. G. Gold, and J. F. Gibbons, *J.Appl. Phys.*, **51**, 274, 1980]. Calculate the maximum steady-state surface temperature associated with irradiation of a material of thermal conductivity $k = 27$ W/m · K and absorptivity $\alpha = 0.45$ by a Gaussian beam with $r_b = 0.1$ mm and power $P = 1$ W. Compare your result with the maximum temperature that would occur if the irradiation was from a circular beam of the same diameter and power, but characterized by a uniform fluence (a *flat* beam). Also calculate the average temperature of the irradiated surface for the uniform fluence case. The temperature far from the irradiated spot is $T_2 = 25$°C.

4.19 Laser beams are used to thermally process materials in a wide range of applications. Often, the beam is scanned along the surface of the material in a desired pattern. Consider the laser heating process of Problem 4.18, except now the laser beam scans the material at a scanning velocity of U. A dimensionless maximum surface temperature can be well correlated by an expression of the form [Nissin, Y. I., A. Lietoila, R. G. Gold, and J. F. Gibbons, *J. Appl. Phys.*, **51**, 274, 1980]

$$\frac{T_{1,\max,U=0} - T_2}{T_{1,\max,U\neq 0} - T_2} = 1 + 0.301Pe - 0.0108Pe^2$$

for the range $0 < Pe < 10$ where Pe is a dimensionless velocity known as the Peclet number. For this problem, $Pe = Ur_b/\sqrt{2}\alpha$ where α is the thermal diffusivity of the material. The maximum material temperature does not occur directly below the laser beam, but a lag distance, δ, behind the center of the moving beam. The dimensionless lag distance can be correlated to Pe by [Sheng, I. C., and Y. Chen, *J. Thermal Stresses*, **14**, 129, 1991]

$$\frac{\delta U}{\alpha} = 0.944Pe^{1.55}$$

(a) For the laser beam size and shape, and material of Problem 4.18, determine the laser power required to achieve $T_{1,\max} = 200$°C for $U = 2$ m/s. The density and specific heat of the material are $\rho = 2000$ kg/m³ and $c = 800$ J/kg · K, respectively.

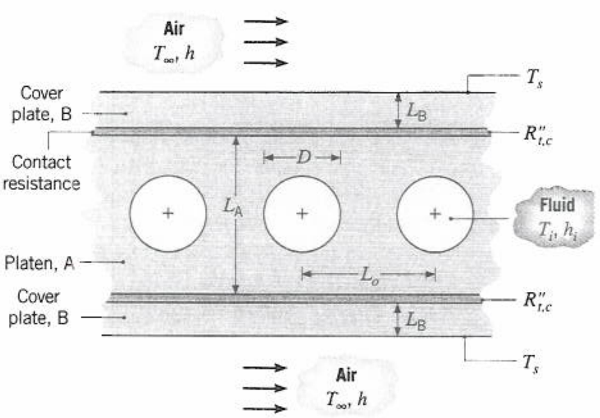
(b) Determine the lag distance, δ, associated with $U = 2$ m/s.

(c) Plot the required laser power to achieve $T_{\max,1} = 200$°C for $0 \leq U \leq 2$ m/s.

Shape Factors with Thermal Circuits

4.20 A cubical glass melting furnace has exterior dimensions of width $W = 5$ m on a side and is constructed from refractory brick of thickness $L = 0.35$ m and thermal conductivity $k = 1.4$ W/m · K. The sides and top of the furnace are exposed to ambient air at 25°C, with free convection characterized by an average coefficient of $h = 5$ W/m^2 · K. The bottom of the furnace rests on a framed platform for which much of the surface is exposed to the ambient air, and a convection coefficient of $h = 5$ W/m^2 · K may be assumed as a first approximation. Under operating conditions for which combustion gases maintain the inner surfaces of the furnace at 1100°C, what is the heat loss from the furnace?

4.21 A hot fluid passes through circular channels of a cast iron platen (A) of thickness $L_A = 30$ mm which is in poor contact with the cover plates (B) of thickness $L_B = 7.5$ mm. The channels are of diameter $D = 15$ mm with a centerline spacing of $L_o = 60$ mm. The thermal conductivities of the materials are $k_A = 20$ W/m · K and $k_B = 75$ W/m · K, while the contact resistance between the two materials is $R''_{t,c} = 2.0 \times 10^{-4}$ m^2 · K/W. The hot fluid is at $T_i = 150$°C, and the convection coefficient is 1000 W/m^2 · K. The cover plate is exposed to ambient air at $T_\infty = 25$°C with a convection coefficient of 200 W/m^2 · K. The shape factor between one channel and the platen top and bottom surfaces is 4.25.

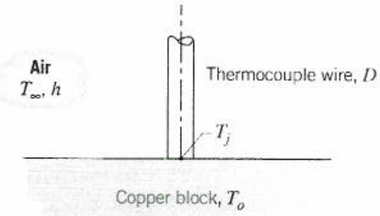


(a) Determine the heat rate from a single channel per unit length of the platen normal to the page, q'_i.

(b) Determine the outer surface temperature of the cover plate, T_s.

(c) Comment on the effects that changing the centerline spacing will have on q'_i and T_s. How would insulating the lower surface affect q'_i and T_s?

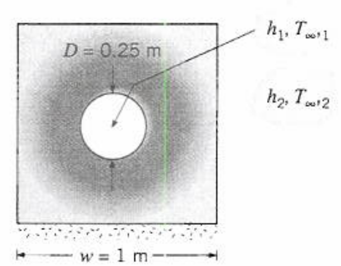
4.22 A long constantan wire of 1-mm diameter is butt welded to the surface of a large copper block, forming a thermocouple junction. The wire behaves as a fin, permitting heat to flow from the surface, thereby depressing the sensing junction temperature T_j below that of the block T_o.



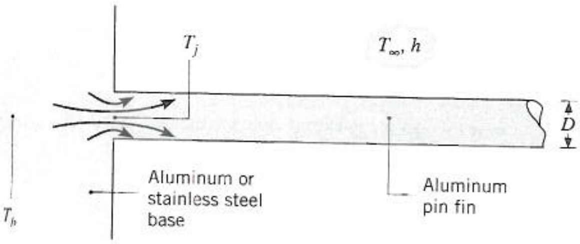
(a) If the wire is in air at 25°C with a convection coefficient of 10 W/m^2 · K, estimate the measurement error $(T_j - T_o)$ for the thermocouple when the block is at 125°C.

(b) For convection coefficients of 5, 10, and 25 W/m^2 · K, plot the measurement error as a function of the thermal conductivity of the block material over the range 15 to 400 W/m · K. Under what circumstances is it advantageous to use smaller diameter wire?

4.23 A hole of diameter $D = 0.25$ m is drilled through the center of a solid block of square cross section with $w = 1$ m on a side. The hole is drilled along the length, $l = 2$ m, of the block, which has a thermal conductivity of $k = 150$ W/m · K. The outer surfaces are exposed to ambient air, with $T_{\infty,2} = 25$°C and $h_2 = 4$ W/m^2 · K, while hot oil flowing through the hole is characterized by $T_{\infty,1} = 300$°C and $h_1 = 50$ W/m^2 · K. Determine the corresponding heat rate and surface temperatures.



4.24 In Chapter 3 we assumed that, whenever fins are attached to a base material, the base temperature is unchanged. What in fact happens is that, if the temperature of the base material exceeds the fluid temperature, attachment of a fin depresses the junction temperature T_j below the original temperature of the base, and heat flow from the base material to the fin is two-dimensional.



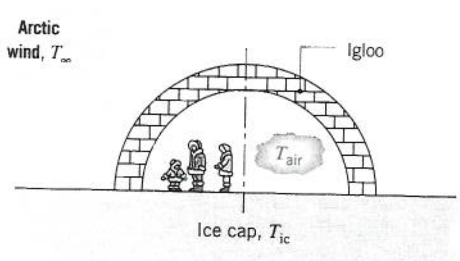
Consider conditions for which a long aluminum pin fin of diameter $D = 5$ mm is attached to a base material whose temperature far from the junction is maintained at $T_b = 100°C$. Fin convection conditions correspond to $h = 50\ W/m^2 \cdot K$ and $T_\infty = 25°C$.

(a) What are the fin heat rate and junction temperature when the base material is (i) aluminum ($k = 240$ W/m · K) and (ii) stainless steel ($k = 15$ W/m · K)?

(b) Repeat the foregoing calculations if a thermal contact resistance of $R''_{t,j} = 3 \times 10^{-5}\ m^2 \cdot K/W$ is associated with the method of joining the pin fin to the base material.

(c) Considering the thermal contact resistance, plot the heat rate as a function of the convection coefficient over the range $10 \leq h \leq 100\ W/m^2 \cdot K$ for each of the two materials.

4.25 An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo the surface heat transfer coefficient is $6\ W/m^2 \cdot K$; on the outside, under normal wind conditions, it is $15\ W/m^2 \cdot K$. The thermal conductivity of compacted snow is 0.15 W/m · K. The temperature of the ice cap on which the igloo sits is −20°C and has the same thermal conductivity as the compacted snow.

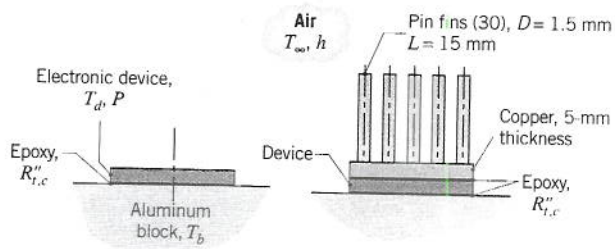


(a) Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside air temperature is $T_\infty = -40°C$. Be sure to consider heat losses through the floor of the igloo.

(b) Using the thermal circuit of part (a), perform a parameter sensitivity analysis to determine which variables have a significant effect on the inside air temperature. For instance, for very high wind conditions, the outside convection coefficient could double or even triple. Does it make sense to construct the igloo with walls half or twice as thick?

4.26 Consider the thin integrated circuit (chip) of Problem 3.136. Instead of attaching the heat sink to the chip surface, an engineer suggests that sufficient cooling might be achieved by mounting the top of the chip onto a large copper ($k = 400$ W/m · K) surface that is located nearby. The metallurgical joint between the chip and the substrate provides a contact resistance of $R''_{t,c} = 5 \times 10^{-6}\ m^2 \cdot K/W$, and the maximum allowable chip temperature is 85°C. If the large substrate temperature is $T_2 = 25°C$ at locations far from the chip, what is the maximum allowable chip power dissipation q_c?

4.27 An electronic device, in the form of a disk 20 mm in diameter, dissipates 100 W when mounted flush on a large aluminum alloy (2024) block whose temperature is maintained at 27°C. The mounting arrangement is such that a contact resistance of $R''_{t,c} = 5 \times 10^{-5}\ m^2 \cdot K/W$ exists at the interface between the device and the block.

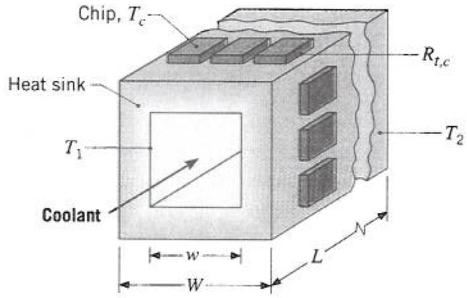


(a) Calculate the temperature the device will reach, assuming that all the power generated by the device must be transferred by conduction to the block.

(b) In order to operate the device at a higher power level, a circuit designer proposes to attach a finned heat sink to the top of the device. The pin fins and base material are fabricated from copper ($k = 400$ W/m · K) and are exposed to an airstream at 27°C for which the convection coefficient is $1000\ W/m^2 \cdot K$. For the device temperature computed in part (a), what is the permissible operating power?

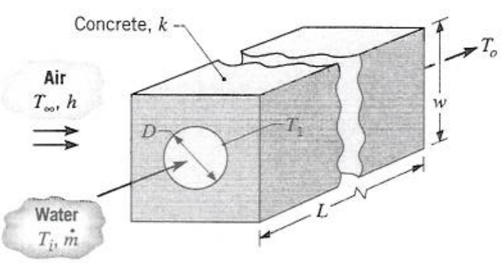
4.28 An aluminum heat sink ($k = 240$ W/m·K) used to cool an array of electronic chips consists of a square channel of inner width $w = 25$ mm, through which liquid flow may be assumed to maintain a uniform surface temperature of

$T_1 = 20°C$. The outer width and length of the channel are $W = 40$ mm and $L = 160$ mm, respectively.



If $N = 120$ chips attached to the outer surfaces of the heat sink maintain an approximately uniform surface temperature of $T_2 = 50°C$ and all of the heat dissipated by the chips is assumed to be transferred to the coolant, what is the heat dissipation per chip? If the contact resistance between each chip and the heat sink is $R_{t,c} = 0.2$ K/W, what is the chip temperature?

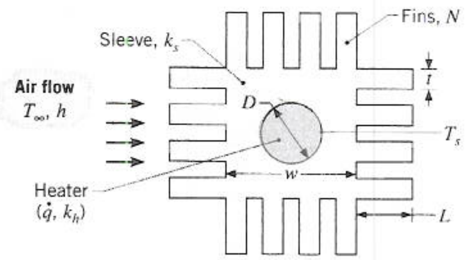
4.29 Hot water is transported from a cogeneration power station to commercial and industrial users through steel pipes of diameter $D = 150$ mm, with each pipe centered in concrete ($k = 1.4$ W/m · K) of square cross section ($w = 300$ mm). The outer surfaces of the concrete are exposed to ambient air for which $T_\infty = 0°C$ and $h = 25$ W/m^2 · K.



(a) If the inlet temperature of water flowing through the pipe is $T_i = 90°C$, what is the heat loss per unit length of pipe in proximity to the inlet? The temperature of the pipe T_1 may be assumed to be that of the inlet water.

(b) If the difference between the inlet and outlet temperatures of water flowing through a 100-m-long pipe is not to exceed 5°C, estimate the minimum allowable flow rate $\dot{m}$. A value of $c = 4207$ J/kg · K may be used for the specific heat of the water.

4.30 The elemental unit of an air heater consists of a long circular rod of diameter D, which is encapsulated by a finned sleeve and in which thermal energy is generated by Ohmic heating. The N fins of thickness t and length L are integrally fabricated with the square sleeve of width w. Under steady-state operating conditions, the rate of thermal energy generation corresponds to the rate of heat transfer to air flow over the sleeve.



(a) Under conditions for which a uniform surface temperature T_s is maintained around the circumference of the heater and the temperature T_∞ and convection coefficient h of the air flow are known, obtain an expression for the rate of heat transfer per unit length to the air. Evaluate the heat rate for $T_s = 300°C$, $D = 20$ mm, an aluminum sleeve ($k_s = 240$ W/m · K), $w = 40$ mm, $N = 16$, $t = 4$ mm, $L = 20$ mm, $T_\infty = 50°C$, and $h = 500$ W/m^2 · K.

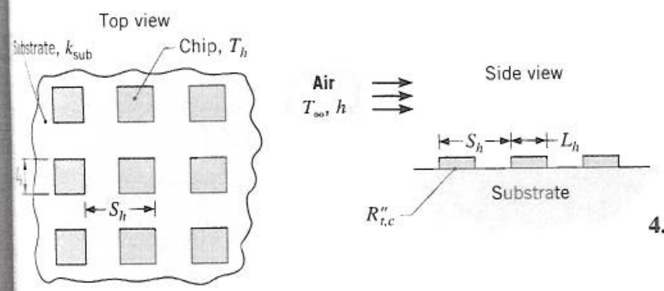
(b) For the foregoing heat rate and a copper heater of thermal conductivity $k_h = 400$ W/m·K, what is the required volumetric heat generation within the heater and its corresponding centerline temperature?

(c) With all other quantities unchanged, explore the effect of variations in the fin parameters (N, L, t) on the heat rate, subject to the constraint that the fin thickness and the spacing between fins cannot be less than 2 mm.

4.31 For a small heat source attached to a large substrate, the *spreading* resistance associated with multidimensional conduction in the substrate may be approximated by the expression (Yovanovich, M. M. and V. W. Antonetti, *Adv. Thermal Modeling Elec. Comp. and Systems*, Vol. 1, A. Bar-Cohen and A. D. Kraus, Eds., Hemisphere, NY, 79–128, 1988)

$$R_{t(\text{sp})} = \frac{1 - 1.410\,A_r + 0.344\,A_r^3 + 0.043\,A_r^5 + 0.034\,A_r^7}{4k_{\text{sub}}A_{s,h}^{1/2}}$$

where $A_r = A_{s,h}/A_{s,\text{sub}}$ is the ratio of the heat source area to the substrate area. Consider application of the expression to an in-line array of square chips of width $L_h = 5$ mm on a side and pitch $S_h = 10$ mm. The interface between the chips and a large substrate of thermal conductivity $k_{\text{sub}} = 80$ W/m·K is characterized by a thermal contact resistance of $R''_{t,c} = 0.5 \times 10^{-4}$ m^2 · K/W.



If a convection heat transfer coefficient of $h = 100\ \text{W/m}^2\cdot\text{K}$ is associated with air flow ($T_\infty = 15°\text{C}$) over the chips and substrate, what is the maximum allowable chip power dissipation if the chip temperature is not to exceed $T_h = 85°\text{C}$?

Finite-Difference Equations: Derivations

4.32 Consider nodal configuration 2 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The horizontal boundary of the internal corner is perfectly insulated and the vertical boundary is subjected to the convection process (T_∞, h).

(b) Both boundaries of the internal corner are perfectly insulated. How does this result compare with Equation 4.41?

4.33 Consider nodal configuration 3 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The boundary is insulated. Explain how Equation 4.42 can be modified to agree with your result.

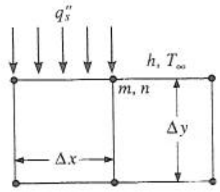
(b) The boundary is subjected to a constant heat flux.

4.34 Consider nodal configuration 4 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The upper boundary of the external corner is perfectly insulated and the side boundary is subjected to the convection process (T_∞, h).

(b) Both boundaries of the external corner are perfectly insulated. How does this result compare with Equation 4.43?

4.35 One of the strengths of numerical methods is their ability to handle complex boundary conditions. In the sketch, the boundary condition changes from specified heat flux, q''_s (into the domain), to convection, at the location of the node m,n. Write the steady-state, two-dimensional finite difference equation at this node.

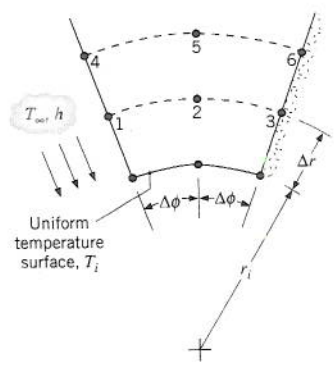


4.36 Consider heat transfer in a one-dimensional (radial) cylindrical coordinate system under steady-state conditions with volumetric heat generation.

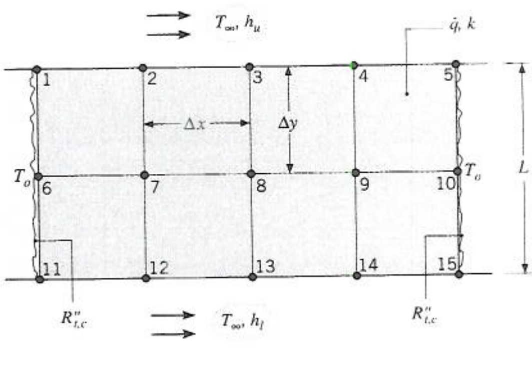
(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for the node n located at the external boundary subjected to the convection process (T_∞, h).

4.37 In a two-dimensional cylindrical configuration, the radial (Δr) and angular ($\Delta\phi$) spacings of the nodes are uniform. The boundary at $r = r_i$ is of uniform temperature T_i. The boundaries in the radial direction are adiabatic (insulated) and exposed to surface convection (T_∞, h), as illustrated. Derive the finite-difference equations for (a) node 2, (b) node 3, and (c) node 1.



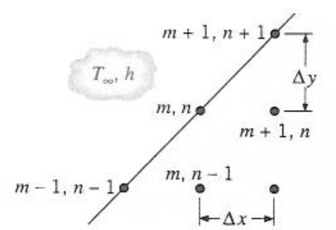
4.38 Upper and lower surfaces of a bus bar are convectively cooled by air at T_∞, with $h_u \neq h_l$. The sides are cooled by maintaining contact with heat sinks at T_o, through a thermal contact resistance of $R''_{t,c}$. The bar is of thermal conductivity k, and its width is twice its thickness L.



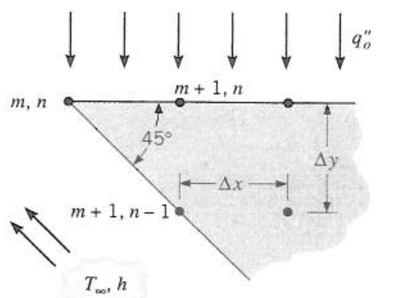
Consider steady-state conditions for which heat is uniformly generated at a volumetric rate $\dot{q}$ due to passage of an electric current. Using the energy balance method, derive finite-difference equations for nodes 1 and 13.

4.39 Derive the nodal finite-difference equations for the following configurations.

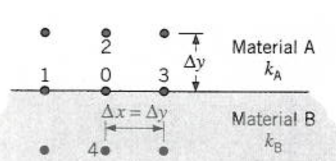
(a) Node m, n on a diagonal boundary subjected to convection with a fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.



(b) Node m, n at the tip of a cutting tool with the upper surface exposed to a constant heat flux q''_o, and the diagonal surface exposed to a convection cooling process with the fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.

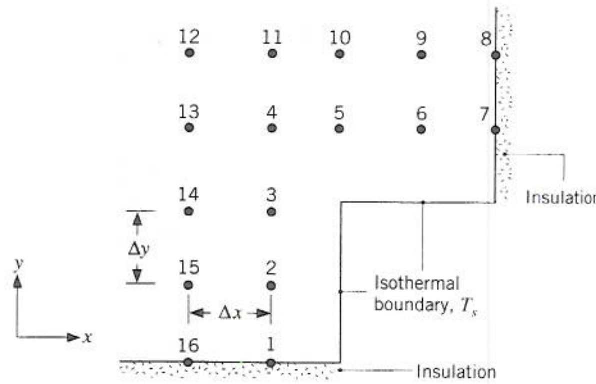


4.40 Consider the nodal point 0 located on the boundary between materials of thermal conductivity k_A and k_B.



Derive the finite-difference equation, assuming no internal generation.

4.41 Consider the two-dimensional grid ($\Delta x = \Delta y$) representing steady-state conditions with no internal volumetric generation for a system with thermal conductivity k. One of the boundaries is maintained at a constant temperature T_s while the others are adiabatic.



Derive an expression for the heat rate per unit length normal to the page crossing the isothermal boundary (T_s).

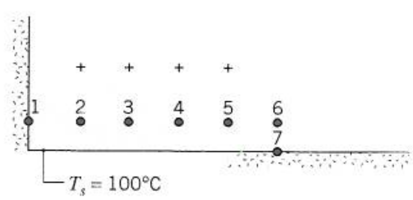
4.42 Consider a one-dimensional fin of uniform cross-sectional area, insulated at its tip, $x = L$. (See Table 3.4, case B). The temperature at the base of the fin T_b and of the adjoining fluid T_∞, as well as the heat transfer coefficient h and the thermal conductivity k, are known.

(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for a node n located at the insulated tip.

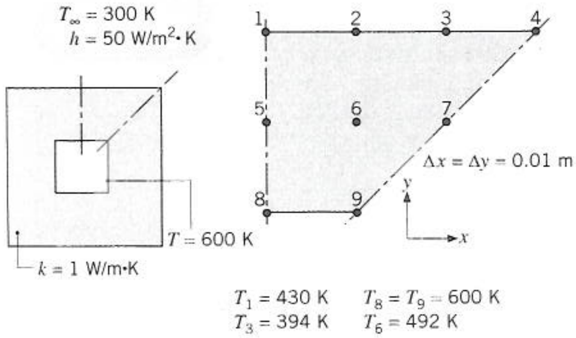
Finite-Difference Equations: Analysis

4.43 Consider the network for a two-dimensional system without internal volumetric generation having nodal temperatures shown below. If the grid space is 125 mm and the thermal conductivity of the material is 50 W/m · K, calculate the heat rate per unit length normal to the page from the isothermal surface (T_s).



Node	T_i (°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

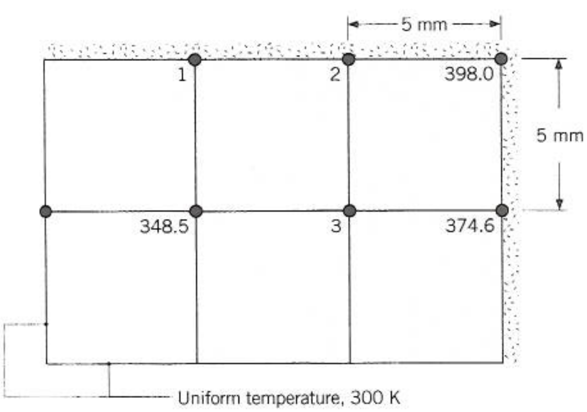
4.44 Consider the square channel shown in the sketch operating under steady-state conditions. The inner surface of the channel is at a uniform temperature of 600 K, while the outer surface is exposed to convection with a fluid at 300 K and a convection coefficient of 50 W/m^2 · K. From a symmetrical element of the channel, a two-dimensional grid has been constructed and the nodes labeled. The temperatures for nodes 1, 3, 6, 8, and 9 are identified.



(a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4, and 7 and determine the temperatures T_2, T_4, and T_7 (K).

(b) Calculate the heat loss per unit length from the channel.

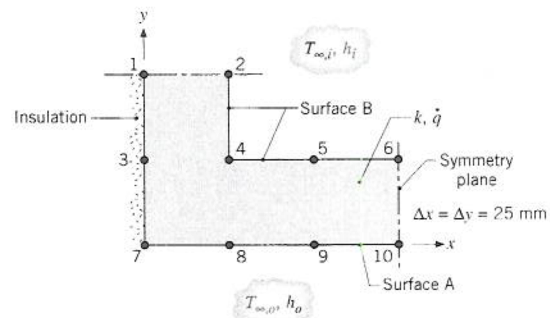
4.45 Steady-state temperatures (K) at three nodal points of a long rectangular rod are as shown. The rod experiences a uniform volumetric generation rate of 5×10^7 W/m^3 and has a thermal conductivity of 20 W/m · K. Two of its sides are maintained at a constant temperature of 300 K, while the others are insulated.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit length (W/m) from the rod using the nodal temperatures. Compare this result with the heat rate calculated from knowledge of the volumetric generation rate and the rod dimensions.

4.46 Steady-state temperatures at selected nodal points of the symmetrical section of a flow channel are known to be T_2 = 95.47°C, T_3 = 117.3°C, T_5 = 79.79°C, T_6 = 77.29°C, T_8 = 87.28°C, and T_{10} = 77.65°C. The wall experiences uniform volumetric heat generation of $\dot{q} = 10^6$ W/m^3 and has a thermal conductivity of k = 10 W/m · K. The inner and outer surfaces of the channel experience convection with fluid temperatures of $T_{\infty,i}$ = 50°C and $T_{\infty,o}$ = 25°C and convection coefficients of h_i = 500 W/m^2·K and h_o = 250 W/m^2 · K.



(a) Determine the temperatures at nodes 1, 4, 7, and 9.

(b) Calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

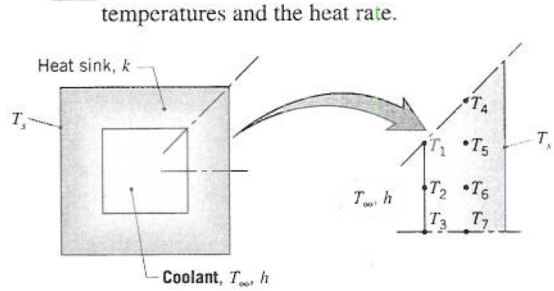
(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

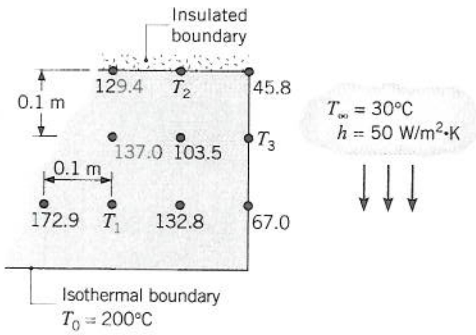
4.47 Consider an aluminum heat sink (k = 240 W/m · K), such as that shown schematically in Problem 4.28. The inner and outer widths of the square channel are w = 20 mm and W = 40 mm, respectively, and an outer surface temperature of T_s = 50°C is maintained by the array of electronic chips. In this case, it is not the inner surface temperature that is known, but conditions (T_∞, h) associated with coolant flow through the channel, and we wish to determine the rate of heat transfer to the coolant per unit length of channel. For this purpose, consider a symmetrical section of the channel and a two-dimensional grid with Δx = Δy = 5 mm.

(a) For T_∞ = 20°C and h = 5000 W/m^2 · K, determine the unknown temperatures, $T_1, \ldots, T_7$, and the rate of heat transfer per unit length of channel, q'.

(b) Assess the effect of variations in h on the unknown temperatures and the heat rate.



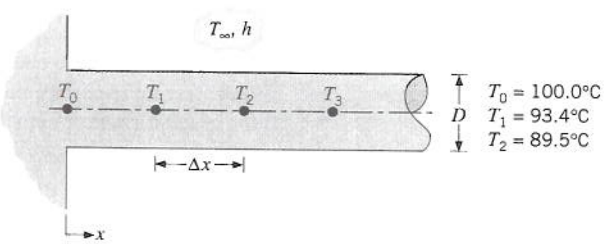
4.48 The steady-state temperatures (°C) associated with selected nodal points of a two-dimensional system having a thermal conductivity of 1.5 W/m · K are shown on the accompanying grid.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit thickness normal to the page from the system to the fluid.

4.49 A steady-state, finite-difference analysis has been performed on a cylindrical fin with a diameter of 12 mm and a thermal conductivity of 15 W/m·K. The convection process is characterized by a fluid temperature of 25°C and a heat transfer coefficient of 25 W/m²·K.

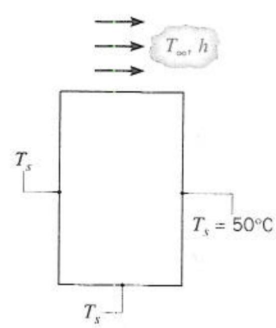


(a) The temperatures for the first three nodes, separated by a spatial increment of x = 10 mm, are given in the sketch. Determine the fin heat rate.

(b) Determine the temperature at node 3, T_3.

Solving the Finite-Difference Equations

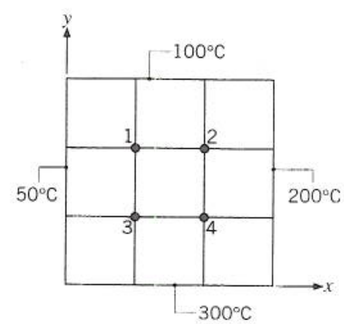
4.50 A long bar of rectangular cross section is 60 mm by 90 mm on a side and has a thermal conductivity of 1 W/m·K. One surface is exposed to a convection process with air at 100°C and a convection coefficient of 100 W/m² · K, while the remaining surfaces are maintained at 50°C.



(a) Using a grid spacing of 30 mm and the Gauss–Seidel iteration method, determine the nodal temperatures and the heat rate per unit length normal to the page into the bar from the air.

(b) Determine the effect of grid spacing on the temperature field and heat rate. Specifically, consider a grid spacing of 15 mm. For this grid, explore the effect of changes in h on the temperature field and the isotherms.

4.51 Consider two-dimensional, steady-state conduction in a square cross section with prescribed surface temperatures.



(a) Determine the temperatures at nodes 1, 2, 3, and 4. Estimate the midpoint temperature.

(b) Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures. Compare your results with those from the coarser grid.

(c) From the results for the finer grid, plot the 75, 150, and 250°C isotherms.

4.52 Consider a long bar of square cross section (0.8 m to the side) and of thermal conductivity 2 W/m·K. Three of these sides are maintained at a uniform temperature of 300°C. The fourth side is exposed to a fluid at 100°C for which the convection heat transfer coefficient is 10 W/m²·K.

(a) Using an appropriate numerical technique with a grid spacing of 0.2 m, determine the midpoint temperature and heat transfer rate between the bar and the fluid per unit length of the bar.

(b) Reducing the grid spacing by a factor of 2, determine the midpoint temperature and heat transfer rate. Plot the corresponding temperature distribution across the surface exposed to the fluid. Also, plot the 200 and 250°C isotherms.

4.53 A long conducting rod of rectangular cross section (20 mm × 30 mm) and thermal conductivity k = 20 W/m · K experiences uniform heat generation at a rate $\dot{q} = 5 \times 10^7$ W/m^3, while its surfaces are maintained at 300 K.

(a) Using a finite-difference method with a grid spacing of 5 mm, determine the temperature distribution in the rod.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

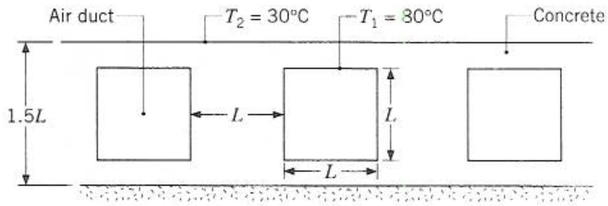
4.54 A flue passing hot exhaust gases has a square cross section, 300 mm to a side. The walls are constructed of refractory brick 150 mm thick with a thermal conductivity of 0.85 W/m · K. Calculate the heat loss from the flue per unit length when the interior and exterior surfaces are maintained at 350 and 25°C, respectively. Use a grid spacing of 75 mm.

4.55 Consider the system of Problem 4.54. The interior surface is exposed to hot gases at 350°C with a convection coefficient of 100 W/m^2 · K, while the exterior surface experiences convection with air at 25°C and a convection coefficient of 5 W/m^2 · K.

(a) Using a grid spacing of 75 mm, calculate the temperature field within the system and determine the heat loss per unit length by convection from the outer surface of the flue to the air. Compare this result with the heat gained by convection from the hot gases to the air.

(b) Determine the effect of grid spacing on the temperature field and heat loss per unit length to the air. Specifically, consider a grid spacing of 25 mm and plot appropriately spaced isotherms on a schematic of the system. Explore the effect of changes in the convection coefficients on the temperature field, and heat loss.

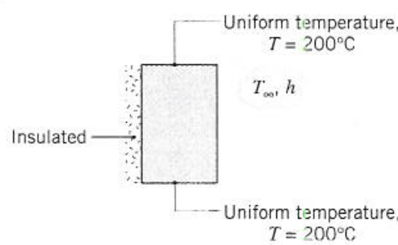
4.56 A common arrangement for heating a large surface area is to move warm air through rectangular ducts below the surface. The ducts are square and located midway between the top and bottom surfaces that are exposed to room air and insulated, respectively.



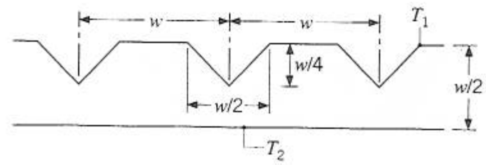
For the condition when the floor and duct temperatures are 30 and 80°C, respectively, and the thermal conductivity of concrete is 1.4 W/m · K, calculate the heat rate from each duct, per unit length of duct. Use a grid spacing with $\Delta x = 2\,\Delta y$, where $\Delta y = 0.125L$ and L = 150 mm.

4.57 Consider the gas turbine cooling scheme of Example 4.4. In Problem 3.23, advantages associated with applying a *thermal barrier coating* (TBC) to the exterior surface of a turbine blade are described. If a 0.5-mm-thick zirconia coating (k=1.3 W/m · K, $R''_{t,c} = 10^{-4}$ m^2 · K/W) is applied to the outer surface of the air-cooled blade, determine the temperature field in the blade for the operating conditions of Example 4.4.

4.58 A long bar of rectangular cross section, 0.4 m × 0.6 m on a side and having a thermal conductivity of 1.5 W/m · K, is subjected to the boundary conditions shown below. Two of the sides are maintained at a uniform temperature of 200°C. One of the sides is adiabatic, and the remaining side is subjected to a convection process with T_∞ = 30°C and h = 50 W/m^2 · K. Using an appropriate numerical technique with a grid spacing of 0.1 m, determine the temperature distribution in the bar and the heat transfer rate between the bar and the fluid per unit length of the bar.



4.59 The top surface of a plate, including its grooves, is maintained at a uniform temperature of T_1 = 200°C. The lower surface is at T_2 = 20°C, the thermal conductivity is 15 W/m · K, and the groove spacing is 0.16 m.

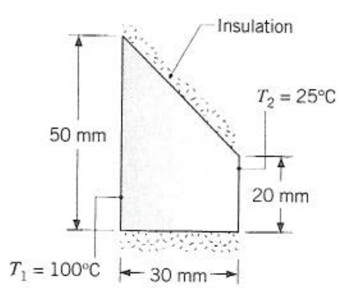


(a) Using a finite-difference method with a mesh size of $\Delta x = \Delta y = 40$ mm, calculate the unknown nodal temperatures and the heat transfer rate per width of groove spacing (w) and per unit length normal to the page.

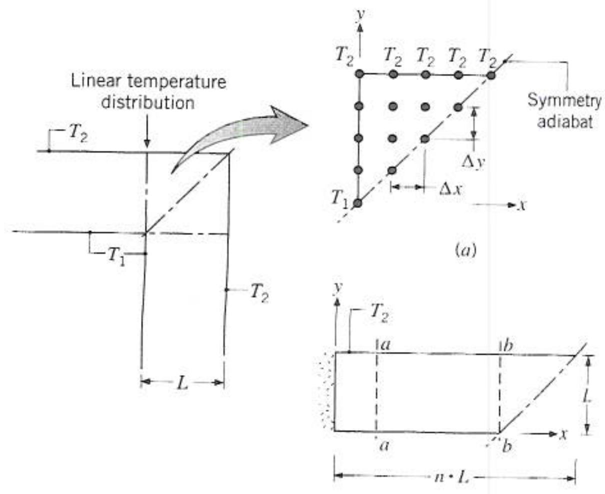
(b) With a mesh size of $\Delta x = \Delta y = 10$ mm, repeat the foregoing calculations, determining the temperature field and the heat rate. Also, consider conditions for which the bottom surface is not at a uniform temperature T_2 but is exposed to a fluid at $T_\infty = 20°\text{C}$. With $\Delta x = \Delta y = 10$ mm, determine the temperature field and heat rate for values of $h = 5$, 200, and 1000 $\text{W/m}^2 \cdot \text{K}$, as well as for $h \rightarrow \infty$.

4.60 Refer to the two-dimensional rectangular plate of Problem 4.2. Using an appropriate numerical method with $\Delta x = \Delta y = 0.25$ m, determine the temperature at the midpoint (1, 0.5).

4.61 A long trapezoidal bar is subjected to uniform temperatures on two surfaces, while the remaining surfaces are well insulated. If the thermal conductivity of the material is 20 W/m·K, estimate the heat transfer rate per unit length of the bar using a finite-difference method. Use the Gauss–Seidel method of solution with a space increment of 10 mm.



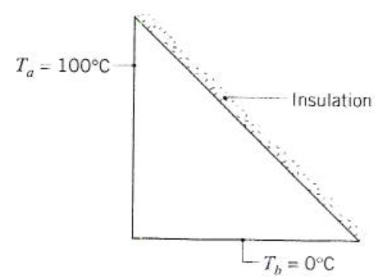
4.62 The shape factor for conduction through the edge of adjoining walls for which $D > L/5$, where D and L are the wall depth and thickness, respectively, is shown in Table 4.1. The two-dimensional symmetrical element of the edge, which is represented by inset (a), is bounded by the diagonal symmetry adiabat and a section of the wall thickness over which the temperature distribution is assumed to be linear between T_1 and T_2.



(a) Using the nodal network of inset (a) with $L = 40$ mm, determine the temperature distribution in the element for $T_1 = 100°\text{C}$ and $T_2 = 0°\text{C}$. Evaluate the heat rate per unit depth ($D = 1$ m) if $k = 1$ W/m·K. Determine the corresponding shape factor for the edge and compare your result with that from Table 4.1.

(b) Choosing a value of $n = 1$ or $n = 1.5$, establish a nodal network for the trapezoid of inset (b) and determine the corresponding temperature field. Assess the validity of assuming linear temperature distributions across sections a–a and b–b.

4.63 The diagonal of a long triangular bar is well insulated, while sides of equivalent length are maintained at uniform temperatures T_a and T_b.

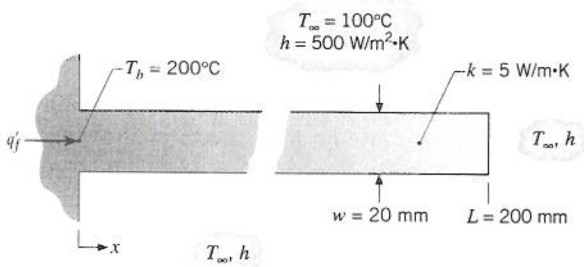


(a) Establish a nodal network consisting of five nodes along each of the sides. For one of the nodes on the diagonal surface, define a suitable control volume and derive the corresponding finite-difference equation. Using this form for the diagonal nodes and appropriate equations for the interior nodes, find the temperature distribution for the bar. On a scale drawing of the shape, show the 25, 50, and 75°C isotherms.

(b) An alternate and simpler procedure to obtain the finite-difference equations for the diagonal nodes follows from recognizing that the insulated diagonal surface is a symmetry plane. Consider a square 5×5 nodal network and represent its diagonal as a symmetry line. Recognize which nodes on either side of the diagonal have identical temperatures. Show that you can treat the diagonal nodes as "interior" nodes and write the finite-difference equations by inspection.

Finite-Element Solutions

4.64 A straight fin of uniform cross section is fabricated from a material of thermal conductivity $k = 5\,\text{W/m}\cdot\text{K}$, thickness $w = 20\,\text{mm}$, and length $L = 200\,\text{mm}$. The fin is very long in the direction normal to the page. The base of the fin is maintained at $T_b = 200°\text{C}$, and the tip condition allows for convection (case A of Table 3.4), with $h = 500\,\text{W/m}^2\cdot\text{K}$ and $T_\infty = 25°\text{C}$.



(a) Assuming one-dimensional heat transfer in the fin, calculate the fin heat rate, q'_f (W/m), and the tip temperature T_L. Calculate the Biot number for the fin to determine whether the one-dimensional assumption is valid.

(b) Using the finite-element method of *FEHT*, perform a two-dimensional analysis on the fin to determine the fin heat rate and tip temperature. Compare your results with those from the one-dimensional, analytical solution of part (a). Use the *View/Temperature Contours* option to display isotherms, and discuss key features of the corresponding temperature field and heat flow pattern. *Hint*: In drawing the outline of the fin, take advantage of symmetry. Use a fine mesh near the base and a coarser mesh near the tip. Why?

(c) Validate your *FEHT* model by comparing predictions with the analytical solution for a fin with thermal conductivities of $k = 50\,\text{W/m}\cdot\text{K}$ and $500\,\text{W/m}\cdot\text{K}$. Is the one-dimensional heat transfer assumption valid for these conditions?

4.65 Consider the long rectangular bar of Problem 4.50 with the prescribed boundary conditions.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the bar to the air stream.

(c) Explore the effect on the heat rate of increasing the convection coefficient by factors of two and three. Explain why the change in the heat rate is not proportional to the change in the convection coefficient.

4.66 Consider the long rectangular rod of Problem 4.53, which experiences uniform heat generation while its surfaces are maintained at a fixed temperature.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

4.67 Consider the symmetrical section of the flow channel of Problem 4.46, with the prescribed values of $\dot{q}$, k, $T_{\infty,i}$, $T_{\infty,o}$, h_i and h_o. Use the finite-element method of *FEHT* to obtain the following results.

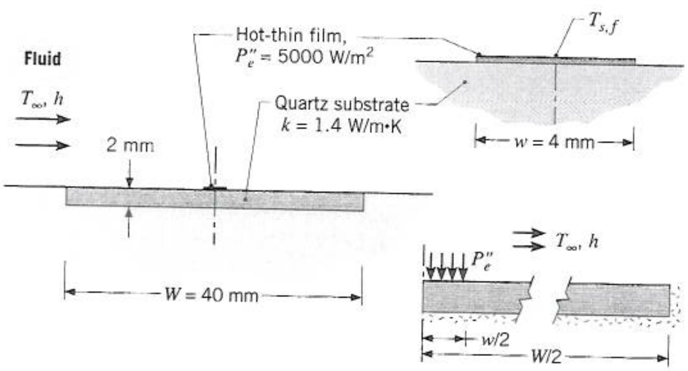
(a) Determine the temperature distribution in the symmetrical section, and use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the temperature distribution, including the hottest and coolest regions and the region with the steepest gradients. Describe the heat flow field.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

4.68 The hot-film heat flux gage shown schematically may be used to determine the convection coefficient of an adjoining fluid stream by measuring the electric power dissipation per unit area, $P''_e\,(\text{W/m}^2)$, and the average surface temperature, $T_{s,f}$, of the film. The power dissipated in the film is transferred directly to the fluid by convection, as well as by conduction into the substrate. If substrate conduction is negligible, the gage measurements can be used to determine the convection coefficient without application of a correction factor. Your assignment is to perform a two-dimensional, steady-state conduction analysis to estimate the fraction of the power dissipation that is conducted into a 2-mm-thick quartz substrate of width $W = 40\,\text{mm}$ and thermal conductivity $k = 1.4\,\text{W/m}\cdot\text{K}$. The thin, hot-film gage has a width of $w = 4\,\text{mm}$ and operates at a uniform power dissipation of $5000\,\text{W/m}^2$. Consider cases for which the fluid temperature is 25°C and the convection coefficient is 500, 1000, and $2000\,\text{W/m}^2\cdot\text{K}$.

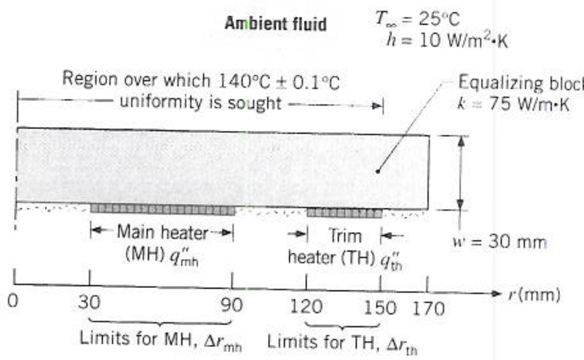


Use the finite-element method of *FEHT* to analyze a symmetrical half-section of the gage and the quartz substrate. Assume that the lower and end surfaces of the substrate are perfectly insulated, while the upper surface experiences convection with the fluid.

(a) Determine the temperature distribution and the conduction heat rate into the region below the hot film for the three values of h. Calculate the fractions of electric power dissipation represented by these rates. *Hint*: Use the *View/Heat Flow* command to find the heat rate across the boundary elements.

(b) Use the *View/Temperature Contours* command to view the isotherms and heat flow patterns. Describe the heat flow paths, and comment on features of the gage design that influence the paths. What limitations on applicability of the gage have been revealed by your analysis?

4.69 A semiconductor industry roadmap for microlithography processing requires that a 300-mm-diameter silicon wafer be maintained at a steady-state temperature of 140°C to within a uniformity of 0.1°C. The design of a hot-plate tool to hopefully meet this requirement is shown schematically. An *equalizing block* (EB), on which the wafer would be placed, is fabricated from an aluminum alloy of thermal conductivity $k = 75\,\text{W/m}\cdot\text{K}$ and is heated by two ring-shaped electrical heaters. The two-zone heating arrangement allows for independent control of a main heater (MH) and a trim heater (TH), which is used to improve the uniformity of the surface temperature for the EB. Your assignment is to size the heaters, MH and TH, by specifying their applied heat fluxes, q''_{mh} and $q''_{th}(\text{W/m}^2)$, and their radial extents, Δr_{mh} and Δr_{th}. The constraints on radial positioning of the heaters are imposed by manufacturing considerations and are shown in the schematic.



Use the finite-element method of *FEHT* to perform a conduction analysis on an axisymmetric EB of 340-mm diameter. The upper and lateral surfaces are exposed to the ambient fluid at $T_\infty = 25$°C with a convection coefficient of $10\,\text{W/m}^2\cdot\text{K}$. The lower surface of the EB is adiabatic, except for the ring sectors with the uniform applied heat fluxes, q''_{mh} and q''_{th}.

(a) For an upper surface of 140°C, perform an overall energy balance on the EB to obtain an initial estimate for the applied heater fluxes. Assume that $q''_{mh} = q''_{th}$ and that each heater extends fully over the radial limits indicated schematically. Use this estimate as a boundary condition in your *FEHT* model; determine the temperature distribution; and using the *View/Temperature Contours* command, examine the isotherms and the temperature distribution across the upper surface of the EB. Did you achieve the desired uniformity?

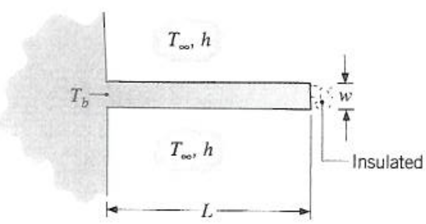
(b) Rerun your *FEHT* model with different values of the heater fluxes, until you obtain the best uniformity possible within the imposed constraints. *Hint*: If you want to know the specific nodal temperatures, see the *View/Tabular Output* page. You may obtain a plot of the surface temperature distribution by highlighting the appropriate nodal data, using the *Copy* command, and pasting the data into *Excel* for graphing.

(c) In what manner would a nonuniform distribution of the convection coefficient across the upper surface of the EB affect the temperature uniformity? For the downward flowing gas stream used in the microlithography process, a representative distribution of the convection coefficient on the upper surface of the EB is $h(r) = h_o[1+a(r/r_o)^n]$, where $h_o = 5.4\,\text{W/m}^2\cdot\text{K}$ and $a = n = 1.5$. For this distribution and retention of a value of $h = 10\,\text{W/m}^2\cdot\text{K}$ at the lateral surface of the EB, can you adjust the trim heater flux to obtain improved uniformity of the surface temperature?

(d) What changes to the design would you propose for improving the surface temperature uniformity?

Special Applications

4.70 A straight fin of uniform cross section is fabricated from a material of thermal conductivity 50 W/m·K, thickness $w = 6$ mm, and length $L = 48$ mm, and is very long in the direction normal to the page. The convection heat transfer coefficient is 500 W/m²·K with an ambient air temperature of $T_\infty = 30°\text{C}$. The base of the fin is maintained at $T_b = 100°\text{C}$, while the fin tip is well insulated.

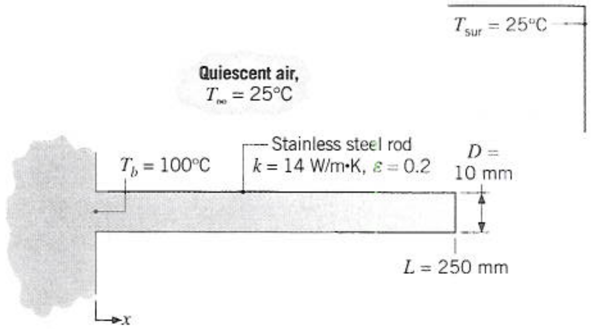


(a) Using a finite-difference method with a space increment of 4 mm, estimate the temperature distribution within the fin. Is the assumption of one-dimensional heat transfer reasonable for this fin?

(b) Estimate the fin heat transfer rate per unit length normal to the page. Compare your result with the one-dimensional, analytical solution, Equation 3.76.

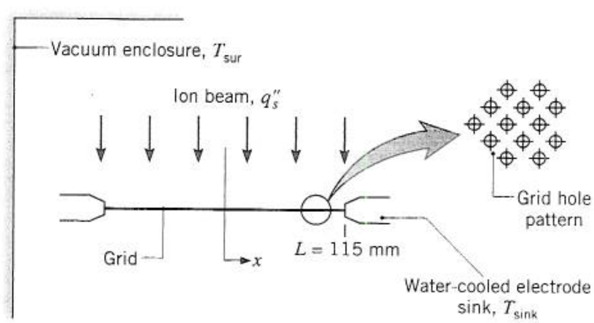
(c) Using the finite-difference mesh of part (a), compute and plot the fin temperature distribution for values of $h = 10$, 100, 500, and 1000 W/m²·K. Determine and plot the fin heat transfer rate as a function of h.

4.71 A rod of 10-mm diameter and 250-mm length has one end maintained at 100°C. The surface of the rod experiences free convection with the ambient air at 25°C and a convection coefficient that depends on the difference between the temperature of the surface and the ambient air. Specifically, the coefficient is prescribed by a correlation of the form, $h_{fc} = 2.89[0.6 + 0.624(T - T_\infty)^{1/6}]^2$, where the units are h_{fc} (W/m²·K) and T (K). The surface of the rod has an emissivity $\varepsilon = 0.2$ and experiences radiation exchange with the surroundings at $T_{sur} = 25°\text{C}$. The fin tip also experiences free convection and radiation exchange.



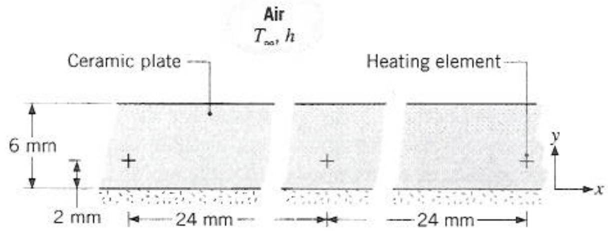
Assuming one-dimensional conduction and using a finite-difference method representing the fin by five nodes, estimate the temperature distribution for the fin. Determine also the fin heat rate and the relative contributions of free convection and radiation exchange. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node. Similarly, for the convection rate equation associated with each node, the free convection coefficient h_{fc} must be evaluated for each node.

4.72 A thin metallic foil of thickness 0.25 mm with a pattern of extremely small holes serves as an acceleration grid to control the electrical potential of an ion beam. Such a grid is used in a chemical vapor deposition (CVD) process for the fabrication of semiconductors. The top surface of the grid is exposed to a uniform heat flux caused by absorption of the ion beam, $q_s'' = 600\,\text{W/m}^2$. The edges of the foil are thermally coupled to water-cooled sinks maintained at 300 K. The upper and lower surfaces of the foil experience radiation exchange with the vacuum enclosure walls maintained at 300 K. The effective thermal conductivity of the foil material is 40 W/m·K and its emissivity is 0.45.



Assuming one-dimensional conduction and using a finite-difference method representing the grid by ten nodes in the x direction, estimate the temperature distribution for the grid. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node.

4.73 Small diameter electrical heating elements dissipating 50 W/m (length normal to the sketch) are used to heat a ceramic plate of thermal conductivity 2 W/m·K. The upper surface of the plate is exposed to ambient air at 30°C with a convection coefficient of 100 W/m²·K, while the lower surface is well insulated.



(a) Using the Gauss–Seidel method with a grid spacing of $\Delta x = 6$ mm and $\Delta y = 2$ mm, obtain the temperature distribution within the plate.

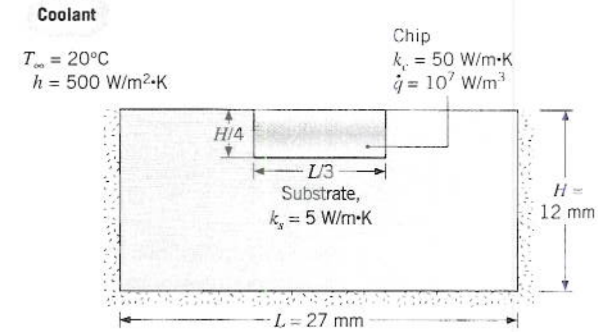
(b) Using the calculated nodal temperatures, sketch four isotherms to illustrate the temperature distribution in the plate.

(c) Calculate the heat loss by convection from the plate to the fluid. Compare this value with the element dissipation rate.

(d) What advantage, if any, is there in not making $\Delta x = \Delta y$ for this situation?

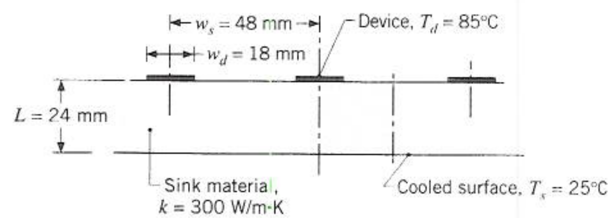
(e) With $\Delta x = \Delta y = 2$ mm, calculate the temperature field within the plate and the rate of heat transfer from the plate. Under no circumstances may the temperature at any location in the plate exceed 400°C. Would this limit be exceeded if the air flow were terminated and heat transfer to the air was by natural convection with $h = 10$ W/m² · K?

4.74 A simplified representation for cooling in very large-scale integration (VLSI) of microelectronics is shown in the sketch. A silicon chip is mounted in a dielectric substrate, and one surface of the system is convectively cooled, while the remaining surfaces are well insulated from the surroundings. The problem is rendered two-dimensional by assuming the system to be very long in the direction perpendicular to the paper. Under steady-state operation, electric power dissipation in the chip provides for uniform volumetric heating at a rate of $\dot{q}$. However, the heating rate is limited by restrictions on the maximum temperature that the chip is allowed to assume.



For the conditions shown on the sketch, will the maximum temperature in the chip exceed 85°C, the maximum allowable operating temperature set by industry standards? A grid spacing of 3 mm is suggested.

4.75 Electronic devices dissipating electrical power can be cooled by conduction to a heat sink. The lower surface of the sink is cooled, and the spacing of the devices w_s, the width of the device w_d, and the thickness L and thermal conductivity k of the heat sink material each affect the thermal resistance between the device and the cooled surface. The function of the heat sink is to *spread* the heat dissipated in the device throughout the sink material.

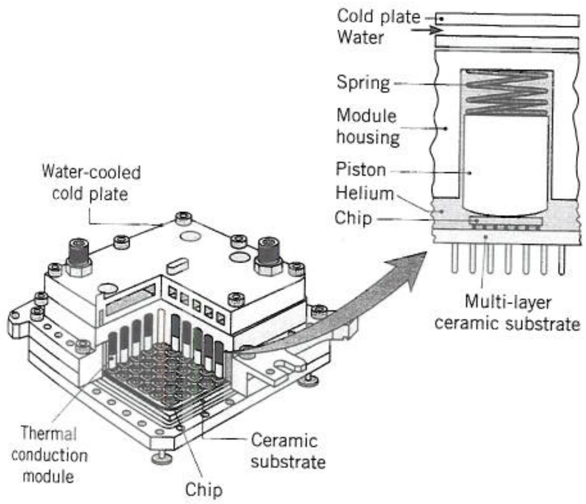


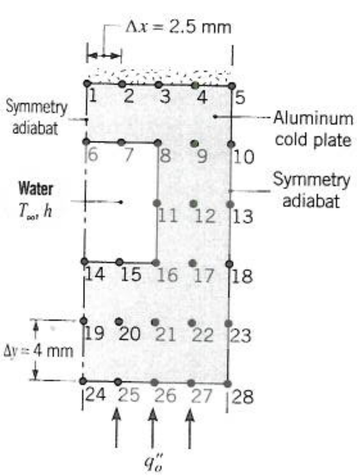
(a) Beginning with the shaded symmetrical element, use a coarse (5 × 5) nodal network to estimate the thermal resistance per unit depth between the device and lower surface of the sink, $R'_{t,d-s}$ (m · K/W). How does this value compare with thermal resistances based on the assumption of one-dimensional conduction in rectangular domains of (i) width w_d and length L and (ii) width w_s and length L?

(b) Using nodal networks with grid spacings three and five times smaller than that in part (a), determine the effect of grid size on the precision of the thermal resistance calculation.

(c) Using the finer nodal network developed for part (b), determine the effect of device width on the thermal resistance. Specifically, keeping w_s and L fixed, find the thermal resistance for values of $w_d/w_s = 0.175, 0.275, 0.375$, and 0.475.

4.76 A major problem in packaging very large-scale integrated (VLSI) circuits concerns cooling of the circuit elements. The problem results from increasing levels of power dissipation within a chip, as well as from packing chips closer together in a module. A novel technique for cooling multichip modules has been developed by IBM. Termed the thermal conduction module (TCM), the chips are soldered to a multilayer ceramic substrate, and heat dissipated in each chip is conducted through a spring-loaded aluminum piston to a water-cooled *cold plate*.

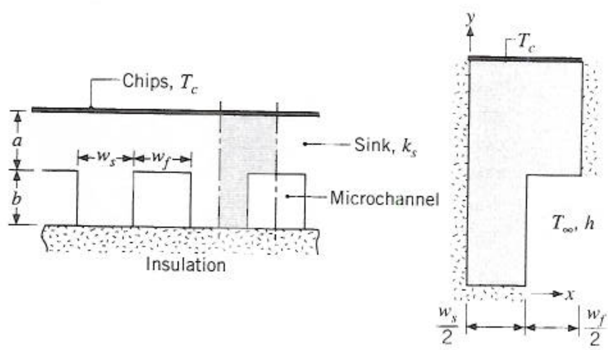




(a) Consider a cold plate fabricated from aluminum ($k = 190\ \text{W/m}\cdot\text{K}$) with regularly spaced rectangular channels through which the water is passed. Under normal operation, power dissipation within the chips results in a uniform heat flux of $q''_o = 10^5\ \text{W/m}^2$ at the base of the cold plate, while water flow provides a temperature of $T_\infty = 15°\text{C}$ and a convection coefficient of $h = 5000\ \text{W/m}^2\cdot\text{K}$ within the channels. We are interested in obtaining the steady-state temperature distribution within the cold plate, and from symmetry considerations, we may confine our attention to the prescribed nodal network. Assuming the top surface of the cold plate to be well insulated, determine the nodal temperatures.

(b) Although there is interest in operating at higher power levels, system reliability considerations dictate that the maximum cold plate temperature must not exceed 40°C. Using the prescribed cold plate geometry and nodal network, assess the effect of changes in operating or design conditions intended to increase the operating heat flux q''_o. Estimate the upper limit for the heat flux.

4.77 A heat sink for cooling computer chips is fabricated from copper ($k_s = 400\ \text{W/m}\cdot\text{K}$), with machined microchannels passing a cooling fluid for which $T_\infty = 25°\text{C}$ and $h = 30{,}000\ \text{W/m}^2\cdot\text{K}$. The lower side of the sink experiences no heat removal, and a preliminary heat sink design calls for dimensions of $a = b = w_s = w_f = 200\ \mu\text{m}$. A symmetrical element of the heat path from the chip to the fluid is shown in the inset.



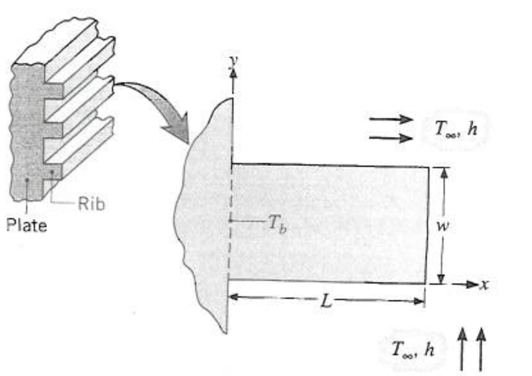
(a) Using the symmetrical element with a square nodal network of $\Delta x = \Delta y = 100\ \mu\text{m}$, determine the corresponding temperature field and the heat rate q' to the coolant per unit channel length (W/m) for a maximum allowable chip temperature $T_{c,\max} = 75°\text{C}$. Estimate the corresponding thermal resistance between the chip surface and the fluid, $R'_{t,c-f}$ (m·K/W). What is the maximum allowable heat dissipation for a chip that measures 10 mm × 10 mm on a side?

(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for the temperature distribution and heat removal rate.

Investigate the effect of grid spacing by considering spatial increments of 50 and 25 μm.

(c) Consistent with the requirement that $a + b = 400\ \mu$m, can the heat sink dimensions be altered in a manner that reduces the overall thermal resistance?

4.78 A plate ($k = 10$ W/m·K) is stiffened by a series of longitudinal ribs having a rectangular cross section with length $L = 8$ mm and width $w = 4$ mm. The base of the plate is maintained at a uniform temperature $T_b = 45°$C, while the rib surfaces are exposed to air at a temperature of $T_\infty = 25°$C and a convection coefficient of $h = 600$ W/m^2·K.

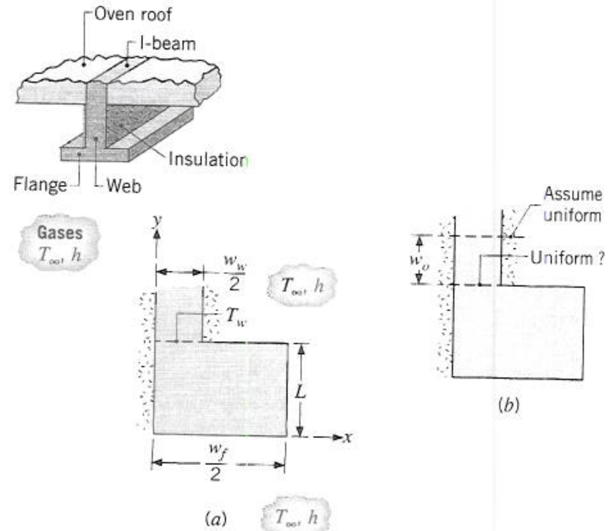


(a) Using a finite-difference method with $\Delta x = \Delta y = 2$ mm and a total of 5×3 nodal points and regions, estimate the temperature distribution and the heat rate from the base. Compare these results with those obtained by assuming that heat transfer in the rib is one-dimensional, thereby approximating the behavior of a fin.

(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for estimates of temperatures and the heat rate. Investigate the effect of grid refinement by reducing the nodal spacing to $\Delta x = \Delta y = 1$ mm (a 9×3 grid) considering symmetry of the center line.

(c) Investigate the nature of two-dimensional conduction in the rib and determine a criterion for which the one-dimensional approximation is reasonable. Do so by extending your finite-difference analysis to determine the heat rate from the base as a function of the length of the rib for the range $1.5 \leq L/w \leq 10$, keeping the length L constant. Compare your results with those determined by approximating the rib as a fin.

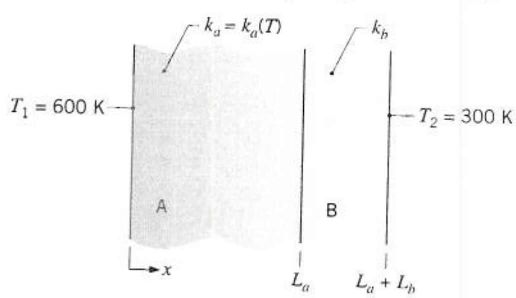
4.79 The bottom half of an I-beam providing support for a furnace roof extends into the heating zone. The web is well insulated, while the flange surfaces experience convection with hot gases at $T_\infty = 400°$C and a convection coefficient of $h = 150$ W/m^2 · K. Consider the symmetrical element of the flange region (inset a), assuming that the temperature distribution across the web is uniform at $T_w = 100°$C. The beam thermal conductivity is 10 W/m · K, and its dimensions are $w_f = 80$ mm, $w_w = 30$ mm, and $L = 30$ mm.



(a) Calculate the heat transfer rate per unit length to the beam using a 5×4 nodal network.

(b) Is it reasonable to assume that the temperature distribution across the web–flange interface is uniform? Consider the L-shaped domain of inset (b) and use a fine grid to obtain the temperature distribution across the web–flange interface. Make the distance $w_o \geq w_w/2$.

4.80 Consider one-dimensional conduction in a plane composite wall. The exposed surfaces of materials A and B are maintained at $T_1 = 600$ K and $T_2 = 300$ K respectively. Material A, of thickness $L_a = 20$ mm, has a temperature-dependent thermal conductivity of $k_a = k_o[1 + \alpha(T - T_o)]$, where $k_o = 4.4$ W/m · K, $\alpha = 0.008\ \text{K}^{-1}$, $T_o = 300$ K and T is in kelvins. Material B is of thickness $L_b = 5$ mm and has a thermal conductivity of $k_b = 1$ W/m · K.

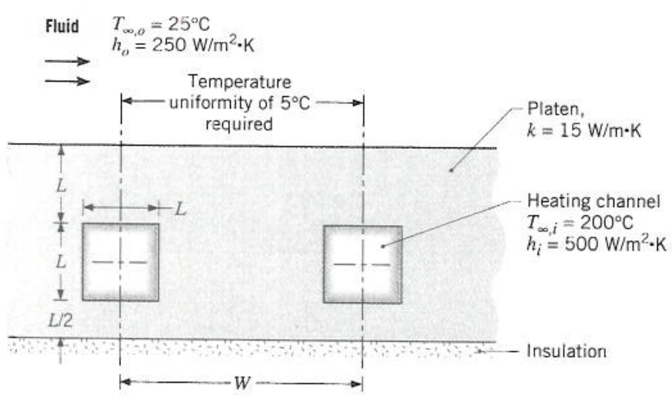


(a) Calculate the heat flux through the composite wall by assuming material A to have a uniform thermal conductivity evaluated at the average temperature of the section.

(b) Using a space increment of 1 mm, obtain the finite-difference equations for the internal nodes and calculate the heat flux considering the temperature-dependent thermal conductivity for Material A. If the *IHT* software is employed, call-up functions from *Tools/Finite-Difference Equations* may be used to obtain the nodal equations. Compare your result with that obtained in part (a).

(c) As an alternative to the finite-difference method of part (b), use the finite-element method of *FEHT* to calculate the heat flux, and compare the result with that from part (a). *Hint*: In the *Specify/Material Properties* box, properties may be entered as a function of temperature (T), the space coordinates (x, y), or time (t). See the *Help* section for more details.

4.81 A platen of thermal conductivity $k = 15\,\text{W/m}\cdot\text{K}$ is heated by flow of a hot fluid through channels of width $L = 20\,\text{mm}$, with $T_{\infty,i} = 200°\text{C}$ and $h_i = 500\,\text{W/m}^2\cdot\text{K}$. The upper surface of the platen is used to heat a process fluid at $T_{\infty,o} = 25°\text{C}$ with a convection coefficient of $h_o = 250\,\text{W/m}^2\cdot\text{K}$. The lower surface of the platen is insulated. To heat the process fluid uniformly, the temperature of the platen's upper surface must be uniform to within 5°C. Use a finite-difference method, such as that of *IHT*, or the finite-element method of *FEHT* to obtain the following results.



(a) Determine the maximum allowable spacing, W, between the channel centerlines that will satisfy the specified temperature uniformity requirement.

(b) What is the corresponding heat rate per unit length from a flow channel?

4.82 Consider the cooling arrangement for the very large-scale integration (VLSI) chip of Problem 4.75. Use the finite-element method of *FEHT* to obtain the following results.

(a) Determine the temperature distribution in the chip-substrate system. Will the maximum temperature exceed 85°C?

(b) Using the *FEHT* model developed for part (a), determine the volumetric heating rate that yields a maximum temperature of 85°C.

(c) What effect would reducing the substrate thickness have on the maximum operating temperature? For a volumetric generation rate of $\dot{q} = 10^7\,\text{W/m}^3$, reduce the thickness of the substrate from 12 mm to 6 mm, keeping all other dimensions unchanged. What is the maximum system temperature for these conditions? What fraction of the chip power generation is removed by convection directly from the chip surface?