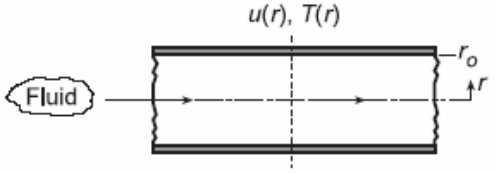
PROBLEM 8.7

KNOWN: Velocity and temperature profiles for laminar flow in a tube of radius $r_o = 10$ mm.

FIND: Mean (or bulk) temperature, T_m, at this axial position.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar incompressible flow, (2) Constant properties.

ANALYSIS: The prescribed velocity and temperature profiles, (m/s and K, respectively) are

$$u(r) = 0.1\,[1-(r/r_o)^2] \qquad T(r) = 344.8 + 75.0\,(r/r_o)^2 - 18.8\,(r/r_o)^4 \tag{1,2}$$

For incompressible flow with constant c_v in a circular tube, from Eq. 8.26, the mean temperature and u_m, the mean velocity, from Eq. 8.8 are, respectively,

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} u(r)\cdot T(r)\cdot r\cdot dr \qquad u_m = \frac{2}{r_o^2}\int_0^{r_o} u(r)\cdot r\cdot dr \tag{3,4}$$

Substituting the velocity profile, Eq. (1), into Eq. (4) and integrating, find

$$u_m = \frac{2}{r_o^2} r_o^2 \int_0^1 0.1\left[1-(r/r_o)^2\right](r/r_o)\,d(r/r_o) = 2\left\{0.1\left[\frac{1}{2}(r/r_o)^2 - \frac{1}{4}(r/r_o)^4\right]\right\}_0^1 = 0.05\,m/s$$

Substituting the profiles and u_m into Eq. (3), find

$$T_m = \frac{2}{(0.05\,m/s)r_o^2} r_o^2 \int_0^1 \left\{0.1\left[1-(r/r_o)^2\right]\right\}\left\{344.8+75.0(r/r_o)^2-18.8(r/r_o)^4\right\}\cdot(r/r_o)\cdot d(r/r_o)$$

$$T_m = 4\int_0^1 \left\{\left[344.8(r/r_o)+75.0(r/r_o)^3-18.8(r/r_o)^5\right]-\left[344.8(r/r_o)^3+75.0(r/r_o)^5-18.8(r/r_o)^7\right]\right\} d(r/r_o)$$

$$T_m = 4\left\{\left[172.40+18.75-3.13\right]-\left[86.20+12.50-2.35\right]\right\} = 367\,K \qquad <$$

The velocity and temperature profiles appear as shown below. Do the values of u_m and T_m found above compare with their respective profiles as you thought? Is the fluid being heated or cooled?