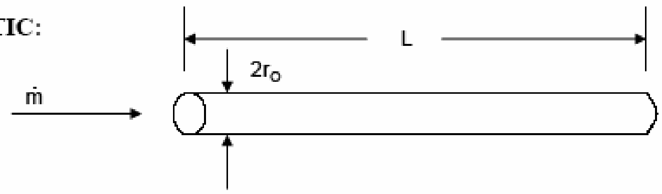
PROBLEM 8.10

KNOWN: Thermal energy equation describing laminar, fully developed flow in a circular pipe with viscous dissipation.

FIND: (a) Left hand side of equation integrated over the pipe volume, (b) viscous dissipation term integrated over the same volume, (c) temperature rise caused by viscous dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Laminar, (3) Fully-developed.

ANALYSIS: (a) The thermal energy equation is given as

$$\rho\, c_p\, u\, \frac{\partial T}{\partial x} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu\left(\frac{du}{dr}\right)^2$$

where u is given by Equation 8.15,

$$u = 2\, u_m\left[1 - (r/r_0)^2\right]$$

Integrating the advection term on the left-hand side over a section of the pipe of length L, we have

$$\text{Adv.} = \int_0^L \int_{A_c} \rho\, c_p\, u\, \frac{\partial T}{dx}\, dA_c\, dx$$

$$= \int_0^L \frac{d}{dx}\left[\int_{A_c} \rho\, c_p\, u\, T\, dA_c\right] dx$$

From Equation 8.25, the term in square brackets is $\dot{m} c_p T_m$, thus

$$\text{Adv.} = \int_0^L \dot{m}\, c_p \frac{dT_m}{dx}\, dx = \dot{m}\, c_p\, (T_{m,o} - T_{m,i})$$ <

which coincides with the right-hand side of Equation 8.34.

(b) Integrating the viscous dissipation term, we have

$$\text{Visc. Diss.} = \int_0^L \int_{A_c} \mu\left(\frac{du}{dr}\right)^2 dA_c\, dx$$

$$= \int_0^L 2\,\pi\,\mu \int_0^{r_o} \left(\frac{du}{dr}\right)^2 r\, dr\, dx$$

$$= 2\,\pi\,\mu\, L \int_0^{r_o} 16\, u_m^2 \frac{r^2}{r_o^4}\, r\, dr$$

$$= 32\,\pi\,\mu\, L\, u_m^2 \left.\frac{r^2}{4r_o^4}\right|_0^{r_o} = 8\,\pi\,\mu\, L\, u_m^2$$ <

Continued….

PROBLEM 8.10 (Cont.)

(c) Using the values from Problem 8.9,

$$\dot{m}\, c_p\ \Delta T_{v.d.} = 8\ \pi\ \mu\ L\ u_m^2$$

$$\Delta T_{v.d.} = 8\ \pi\ \mu\ L\ u_m^2/\dot{m}\ c_p$$

where $u_m = \dot{m}/\rho A_c$. Thus

$$\Delta T_{v.d.} = \frac{8\ \pi\ \mu\ L\ \dot{m}}{\rho^2\ c_p\ A_c^2}$$

$$= \frac{8\pi \times 0.765\ \mathrm{N\cdot s/m^2} \times 100{,}000\ \mathrm{m} \times 500\ \mathrm{kg/s}}{\left[(900\ \mathrm{kg/m^3})^2 \times 2000\ \mathrm{J/kg\cdot K} \times (\pi \times (1.2\ \mathrm{m})^2/4)^2\right]}$$

$$= 0.46°\mathrm{C}$$ <

COMMENTS: (1) Even in the case of a long pipe with a highly viscous fluid, the temperature rise due to viscous dissipation is quite small. (2) The temperature rise due to viscous dissipation is identical to the temperature rise due to flow work in Problem 8.9. This is no coincidence. In fully-developed pipe flow, there is a balance between the viscous forces (friction) and the pressure drop needed to overcome them. As a result, viscous dissipation exactly equals the work done by the pressure forces (flow work). Conservation of energy can be expressed in a form that includes flow work (for example, Equation 1.11d) or in a form that includes viscous dissipation (for example, Equation 6.29), and in the case of fully-developed pipe flow they are equal.