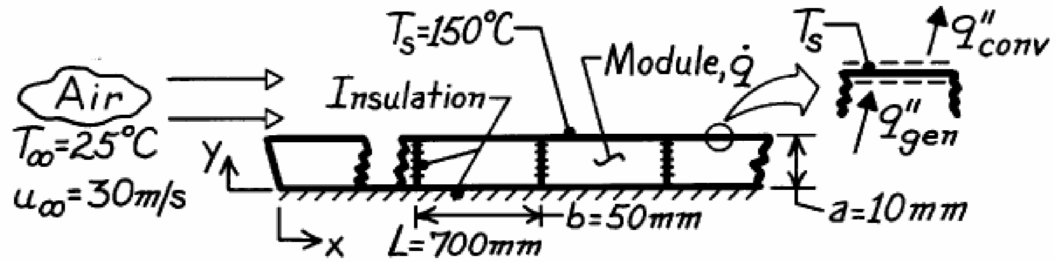
PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): $k = 5.2$ W/m·K, $c_p = 320$ J/kg·K, $\rho = 2300$ kg/m^3; *Table A-4*, Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360 \text{ K, 1 atm}\right)$: $k = 0.0308$ W/m·K, $\nu = 22.02 \times 10^{-6}$ m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}(T_s - T_\infty) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30 \text{ m/s} \times 0.70 \text{ m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x \;(L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30 \text{ m/s} \times (0.700 + 0.050/2)\text{m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5}(0.698)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308 \text{ W/m}\cdot\text{K}}{0.725} = 69.7 \text{ W/m}^2\cdot\text{K}.$$

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Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2\cdot\text{K}(150-25)\text{K}}{0.010\ \text{m}} = 8.713\times10^5\ \text{W/m}^3. \qquad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713\times10^5\ \text{W/m}^3\times(0.010\ \text{m})^2}{2\times5.2\ \text{W/m}\cdot\text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \qquad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{module} = q_{0\to L+b} - q_{0\to L}$$

$$\bar{h}\cdot b = \bar{h}_{L+b}\cdot(L+b) - \bar{h}_L\cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b}\frac{L+b}{b} - \bar{h}_L\frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037Re_x^{4/5} - 871\right)Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.79\ \text{W/m}^2\cdot\text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$\bar{h} = \left[54.79\frac{0.750}{0.050} - 53.73\frac{0.700}{0.05}\right]\ \text{W/m}^2\cdot\text{K} = 69.7\ \text{W/m}^2\cdot\text{K}.$$

which is in excellent agreement with the approximate result employed in part (a).