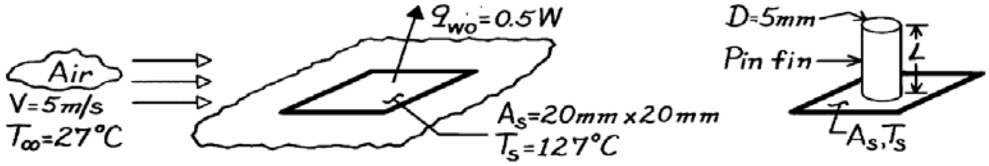
PROBLEM 7.46

KNOWN: Pin fin installed on a surface with prescribed heat rate and temperature.

FIND: (a) Maximum heat removal rate possible, (b) Length of the fin, (c) Effectiveness, ε_f, (d) Percentage increase in heat rate from surface due to fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Conditions over A_s are uniform for both situations, (3) Conditions over fin length are uniform, (4) Flow over pin fin approximates cross-flow.

PROPERTIES: *Table A-4*, Air ($T_f = (T_\infty + T_s)/2 = (27 + 127)°C/2 = 350$ K): $\nu = 20.92 \times 10^{-6}$ m^2/s, $k = 30.0 \times 10^{-3}$ W/m·K, Pr = 0.700. *Table A-1*, SS AISI304 ($\overline{T} = T_f = 350$ K): k = 15.8 W/m·K.

ANALYSIS: (a) Maximum heat rate from fin occurs when fin is infinitely long,

$$q_f = M = \left(\overline{h}PkA_c\right)^{1/2}\theta_b \qquad (1)$$

from Eq. 3.80. Estimate convection heat transfer coefficient for cross-flow over cylinder,

$$Re_D = \frac{VD}{\nu} = 5\text{ m/s}\times0.005\text{ m}/20.92\times10^{-6}\text{m}^2/\text{s} = 1195.$$

Using the Hilpert correlation, Eq. 7.55, with Table 7.2,find

$$\overline{h} = \frac{k}{D}CRe_D^m Pr = \left(0.030\text{W/m}\cdot\text{K}/0.005\text{m}\right)0.683\left(1195\right)^{0.466}\left(0.700\right)^{1/3} = 98.9\text{W/m}^2\cdot\text{K}$$

From Eq. (1), with $P = \pi D$, $A_c = \pi D^2/4$, and $\theta_b = T_s - T_\infty$, find

$$q_f = \left(98.9\text{ W/m}^2\cdot\text{K}\times\pi\left(0.005\text{ m}\right)\times15.8\text{ W/m}\cdot\text{K}\times\pi\left(0.005\text{ m}\right)^2/4\right)^{1/2}\left(127-27\right)\text{K} = 2.20\text{ W.} \qquad <$$

(b) From Example 3.9, $L \approx L_\infty = 2.65(kA_c/hP)^{1/2}$. Hence,

$$L \approx L_\infty = 2.65\left[15.8\text{ W/m}\cdot\text{K}\times\pi\left(0.005\text{ m}\right)^2/4/98.9\text{ W/m}^2\cdot\text{K}\times\pi\left(0.005\text{ m}\right)\right]^{1/2} = 37.4\text{ mm.} \qquad <$$

(c) From Eq. 3.81, with h_s used for the base area A_s, the effectiveness is

$$\varepsilon_f = \frac{q_f}{h_s A_{c,b}\theta_b} = \frac{q_f}{q_{wo}}\frac{A_s}{A_{c,b}} = \frac{2.2\text{ W}}{0.5\text{ W}}\cdot\frac{\left(0.020\times0.020\right)\text{m}^2}{\pi\left(0.005\text{ m}\right)^2/4} = 89.6 \qquad <$$

where $\qquad h_s = q_{wo}/A_s\theta_b$.

(d) The percentage increase in heat rate with the installed fin (w) is

$$\frac{q_w - q_{wo}}{q_{wo}}\times100 = \left(\left[q_f + h_s\left(A_s - \pi D^2/4\right)\left(T_s - T_\infty\right)\right] - q_{wo}\right)\times100/q_{wo}$$

$$\Delta q/q = \left\{\left[2.2\text{ W} + 12.5\text{ W/m}^2\cdot\text{K}\left(\left[0.02\text{ m}\right]^2 - \left(\pi/4\right)\left(0.005\text{ m}\right)^2\right)100\text{ K} - 0.5\text{ W}\right\}\times100/0.5\text{ W}$$

$$\Delta q/q = 435\%. \qquad <$$