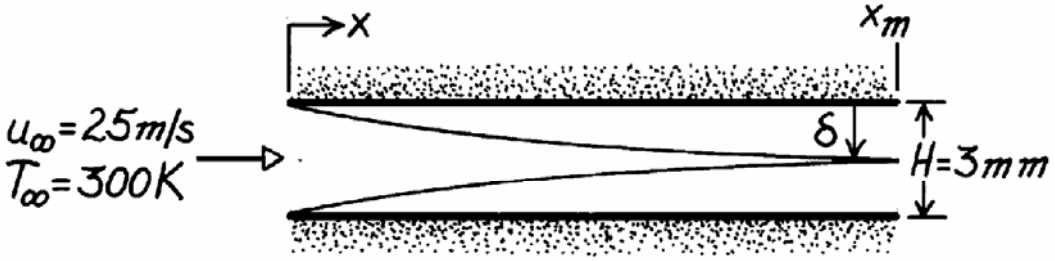
PROBLEM 7.3

KNOWN: Velocity and temperature of air in parallel flow over a flat plate.

FIND: (a) Velocity boundary layer thickness at selected stations. Distance at which boundary layers merge for plates separated by H = 3 mm. (b) Surface shear stress and v(δ) at selected stations.

SCHEMATIC:



ASSUMPTIONS: (1) Steady flow, (2) Boundary layer approximations are valid, (3) Flow is laminar.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\rho = 1.161$ kg/m^3, $\nu = 15.89 \times 10^{-6}$ m^2/s.

ANALYSIS: (a) For laminar flow,

$$\delta = \frac{5x}{Re_x^{1/2}} = \frac{5}{(u_\infty/\nu)^{1/2}} x^{1/2} = \frac{5x^{1/2}}{\left(25\ \text{m/s}/15.89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2}} = 3.99\times10^{-3}\ x^{1/2}.$$

x (m)	0.001	0.01	0.1
δ (mm)	0.126	0.399	1.262

Boundary layer merger occurs at $x = x_m$ when $\delta = 1.5$ mm. Hence

$$x_m^{1/2} = \frac{0.0015\ \text{m}}{3.99\times10^{-3}\ \text{m}^{1/2}} = 0.376\ \text{m}^{1/2} \qquad x_m = 141\ \text{mm}. \quad <$$

(b) The shear stress is

$$\tau_{s,x} = 0.664\frac{\rho u_\infty^2/2}{Re_x^{1/2}} = 0.664\frac{\rho u_\infty^2/2}{(u_\infty/\nu)^{1/2}x^{1/2}} = \frac{0.664\times1.161\ \text{kg/m}^3(25\ \text{m/s})^2/2}{\left(25\ \text{m/s}/15.89\times10^{-6}\ \text{m}^2/\text{s}\right)^{1/2}x^{1/2}} = \frac{0.192}{x^{1/2}}\left(\text{N/m}^2\right).$$

x (m)	0.001	0.01	0.1
$\tau_{s,x}$ (N/m^2)	6.07	1.92	0.61

The velocity distribution in the boundary layer is $v = (1/2)(\nu u_\infty/x)^{1/2}(\eta df/d\eta - f)$. At $y = \delta$, $\eta \approx 5.0$, $f \approx 3.24$, $df/d\eta \approx 0.991$.

$$v = \frac{0.5}{x^{1/2}}\left(15.89\times10^{-6}\ \text{m}^2/\text{s}\times25\ \text{m/s}\right)^{1/2}(5.0\times0.991-3.28) = \left(0.0167/x^{1/2}\right)\text{m/s}.$$

x (m)	0.001	0.01	0.1
v (m/s)	0.528	0.167	0.053

COMMENTS: (1) $v << u_\infty$ and $\delta << x$ are consistent with BL approximations. Note, $v \to \infty$ as $x \to 0$ and approximations breakdown very close to the leading edge. (2) Since $Re_{x_m} = 2.22\times10^5$, laminar BL model is valid. (3) Above expressions are approximations for flow between parallel plates, since $du_\infty/dx > 0$ and $dp/dx < 0$.