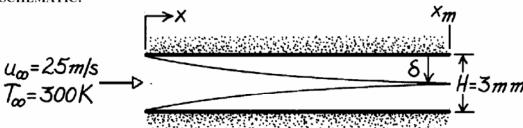
PROBLEM 7.3

KNOWN: Velocity and temperature of air in parallel flow over a flat plate.

FIND: (a) Velocity boundary layer thickness at selected stations. Distance at which boundary layers merge for plates separated by H = 3 mm. (b) Surface shear stress and $v(\delta)$ at selected stations.

SCHEMATIC:



ASSUMPTIONS: (1) Steady flow, (2) Boundary layer approximations are valid, (3) Flow is laminar.

PROPERTIES: *Table A-4*, Air (300 K, 1 atm): $\rho = 1.161 \text{ kg/m}^3$, $v = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$.

ANALYSIS: (a) For laminar flow,

$$\begin{split} \delta &= \frac{5x}{\text{Re}_x^{1/2}} = \frac{5}{\left(u_\infty / v\right)^{1/2}} x^{1/2} = \frac{5x^{1/2}}{\left(25 \text{ m/s/}15.89 \times 10^{-6} \text{ m}^2 / \text{s}}\right)^{1/2}} = 3.99 \times 10^{-3} \text{ x}^{1/2}. \\ x \text{ (m)} & 0.001 & 0.01 & 0.1 \\ \delta \text{ (mm)} & 0.126 & 0.399 & 1.262 \end{split}$$

Boundary layer merger occurs at $x = x_m$ when $\delta = 1.5$ mm. Hence

$$x_{\rm m}^{1/2} = \frac{0.0015 \text{ m}}{3.99 \times 10^{-3} \text{ m}^{1/2}} = 0.376 \text{ m}^{1/2}$$
 $x_{\rm m} = 141 \text{ mm}.$

(b) The shear stress is

(b) The shear stress is
$$\tau_{s,x} = 0.664 \frac{\rho u_{\infty}^2/2}{Re_x^{1/2}} = 0.664 \frac{\rho u_{\infty}^2/2}{\left(u_{\infty}/\nu\right)^{1/2} x^{1/2}} = \frac{0.664 \times 1.161 \text{ kg/m}^3 \left(25 \text{ m/s}\right)^2/2}{\left(25 \text{ m/s/15.89} \times 10^{-6} \text{ m}^2/\text{s}\right)^{1/2} x^{1/2}} = \frac{0.192}{x^{1/2}} \left(N/m^2\right).$$

$$x \text{ (m)} \qquad 0.001 \quad 0.01 \quad 0.1 \\ \tau_{s,x} \left(N/m^2\right) \quad 6.07 \quad 1.92 \quad 0.61$$

The velocity distribution in the boundary layer is $v=(1/2)\left(\nu u \infty/x\right)^{1/2}\left(\eta df/d\eta-f\right)$. At $y=\delta,\,\eta\approx5.0,\,f$

$$\begin{aligned} v &= \frac{0.5}{x^{1/2}} \left(15.89 \times 10^{-6} \text{ m}^2 / \text{s} \times 25 \text{ m/s} \right)^{1/2} \left(5.0 \times 0.991 - 3.28 \right) = \left(0.0167 / x^{1/2} \right) \text{m/s}. \\ x \left(\text{m} \right) & 0.001 & 0.01 & 0.1 \\ v \left(\text{m/s} \right) & 0.528 & 0.167 & 0.053 \end{aligned}$$

COMMENTS: (1) $v \le u_{\infty}$ and $\delta \le x$ are consistent with BL approximations. Note, $v \to \infty$ as $x \to \infty$ 0 and approximations breakdown very close to the leading edge. (2) Since $Re_{X_m} = 2.22 \times 10^5$, laminar BL model is valid. (3) Above expressions are approximations for flow between parallel plates, since $du_{\infty}/dx > 0$ and dp/dx < 0.

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which the textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.