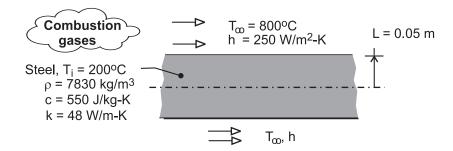
PROBLEM 5.37

KNOWN: Thickness, properties and initial temperature of steel slab. Convection conditions.

FIND: Heating time required to achieve a minimum temperature of 550°C in the slab.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

ANALYSIS: With a Biot number of $hL/k = (250 \text{ W/m}^2 \cdot \text{K} \times 0.05 \text{m})/48 \text{ W/m} \cdot \text{K} = 0.260$, a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, and from the one-term approximation to the transient thermal response of a plane wall, Eq. (5.41), we obtain

$$\theta_{o}^{*} = \frac{T_{o} - T_{\infty}}{T_{i} - T_{\infty}} = \frac{(550 - 800)^{\circ}C}{(200 - 800)^{\circ}C} = 0.417 = C_{1} \exp(-\zeta_{1}^{2} F_{o})$$

With $\zeta_1 \approx 0.488$ rad and $C_1 \approx 1.0396$ from Table 5.1 and $\alpha = k/\rho c = 1.115 \times 10^{-5} \text{ m}^2/\text{s}$,

$$-\zeta_1^2 \left(\alpha t / L^2 \right) = \ln \left(0.401 \right) = -0.914$$

$$t = \frac{0.914L^2}{\zeta_1^2 \alpha} = \frac{0.841(0.05m)^2}{(0.488)^2 1.115 \times 10^{-5} m^2 / s} = 861s$$

COMMENTS: The surface temperature at t = 861s may be obtained from Eq. (5.40b), where $\theta^* = \theta_0^* \cos\left(\zeta_1 x^*\right) = 0.417 \cos\left(0.488 \text{ rad}\right) = 0.368$. Hence, $T\left(L,792s\right) \equiv T_s = T_\infty + 0.368\left(T_i - T_\infty\right) = 800^\circ\text{C} - 221^\circ\text{C} = 579^\circ\text{C}$. Assuming a surface emissivity of $\epsilon = 1$ and surroundings that are at $T_{sur} = T_\infty = 800^\circ\text{C}$, the radiation heat transfer coefficient corresponding to this surface temperature is $h_r = \epsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 205\,\text{W}/\text{m}^2 \cdot \text{K}$. Since this value is comparable to the convection coefficient, radiation is not negligible and the desired heating will occur well before t = 861s.

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