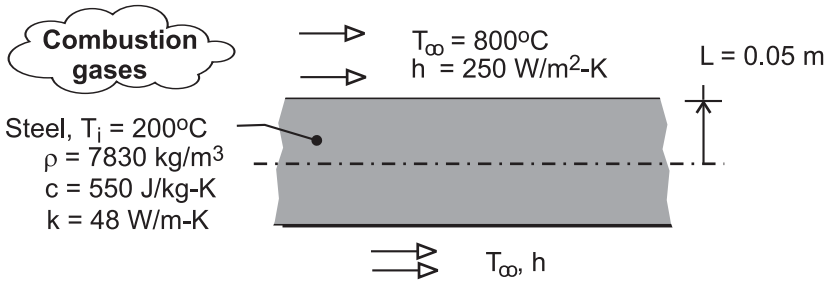
PROBLEM 5.37

KNOWN: Thickness, properties and initial temperature of steel slab. Convection conditions.

FIND: Heating time required to achieve a minimum temperature of 550°C in the slab.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Negligible radiation effects, (3) Constant properties.

ANALYSIS: With a Biot number of hL/k = (250 W/m^2·K × 0.05m)/48 W/m·K = 0.260, a lumped capacitance analysis should not be performed. At any time during heating, the lowest temperature in the slab is at the midplane, and from the one-term approximation to the transient thermal response of a plane wall, Eq. (5.41), we obtain

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{(550-800)°C}{(200-800)°C} = 0.417 = C_1 \exp\left(-\zeta_1^2 Fo\right)$$

With $\zeta_1 \approx 0.488$ rad and $C_1 \approx 1.0396$ from Table 5.1 and $\alpha = k/\rho c = 1.115\times10^{-5} m^2/s$,

$$-\zeta_1^2\left(\alpha t/L^2\right) = \ln(0.401) = -0.914$$

$$t = \frac{0.914 L^2}{\zeta_1^2 \alpha} = \frac{0.841(0.05m)^2}{(0.488)^2 1.115\times10^{-5} m^2/s} = 861s \quad <$$

COMMENTS: The surface temperature at t = 861s may be obtained from Eq. (5.40b), where $\theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right) = 0.417\cos(0.488\ \text{rad}) = 0.368$. Hence, $T(L, 792s) \equiv T_s = T_\infty + 0.368(T_i - T_\infty)$ = 800°C – 221°C = 579°C. Assuming a surface emissivity of ε = 1 and surroundings that are at $T_{sur} = T_\infty = 800°C$, the radiation heat transfer coefficient corresponding to this surface temperature is $h_r = \varepsilon\sigma(T_s + T_{sur})\left(T_s^2 + T_{sur}^2\right) = 205\ W/m^2\cdot K$. Since this value is comparable to the convection coefficient, radiation is not negligible and the desired heating will occur well before t = 861s.