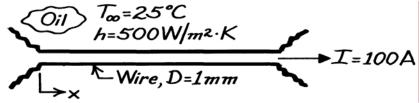
PROBLEM 5.18

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of its steady-state value.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.

PROPERTIES: Wire (given): $\rho = 8000 \text{ kg/m}^3$, $c_p = 500 \text{ J/kg} \cdot \text{K}$, $k = 20 \text{ W/m} \cdot \text{K}$, $R'_e = 0.01 \Omega/m$.

ANALYSIS: Since

Bi =
$$\frac{h(r_0/2)}{k} = \frac{500 \text{ W/m}^2 \cdot K(2.5 \times 10^{-4} \text{m})}{20 \text{ W/m} \cdot \text{K}} = 0.006 < 0.1$$

the lumped capacitance method can be used. The problem has been analyzed in Example 1.3, and without radiation the steady-state temperature is given by

$$\pi \operatorname{Dh}(\mathrm{T}-\mathrm{T}_{\infty}) = \mathrm{I}^{2}\mathrm{R}_{\mathrm{e}}^{\prime}.$$

Hence

$$T = T_{\infty} + \frac{I^2 R'_e}{\pi Dh} = 25^{\circ}C + \frac{(100A)^2 0.01\Omega / m}{\pi (0.001 \text{ m}) 500 \text{ W/m}^2 \cdot \text{K}} = 88.7^{\circ}C.$$

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.3)

$$\frac{\mathrm{dT}}{\mathrm{dt}} = \frac{\mathrm{I}^{2}\mathrm{R}_{\mathrm{e}}^{\prime}}{\rho \mathrm{c}_{\mathrm{p}}\left(\pi\mathrm{D}^{2}/4\right)} - \frac{4\mathrm{h}}{\rho \mathrm{c}_{\mathrm{p}}\mathrm{D}}\left(\mathrm{T}-\mathrm{T}_{\infty}\right).$$

With $T = T_i = 25^{\circ}C$ at t = 0, the solution is

$$\frac{\mathrm{T}-\mathrm{T}_{\infty}-\left(\mathrm{I}^{2}\mathrm{R}_{\mathrm{e}}^{\prime}/\pi \mathrm{Dh}\right)}{\mathrm{T}_{\mathrm{i}}-\mathrm{T}_{\infty}-\left(\mathrm{I}^{2}\mathrm{R}_{\mathrm{e}}^{\prime}/\pi \mathrm{Dh}\right)}=\exp\left(-\frac{4\mathrm{h}}{\rho \mathrm{c}_{\mathrm{p}}\mathrm{D}}\mathrm{t}\right).$$

Substituting numerical values, find

$$\frac{87.7 - 25 - 63.7}{25 - 25 - 63.7} = \exp\left(-\frac{4 \times 500 \text{ W/m}^2 \cdot \text{K}}{8000 \text{ kg/m}^3 \times 500 \text{ J/kg} \cdot \text{K} \times 0.001 \text{ m}} t\right)$$

$$t = 8.31s.$$

COMMENTS: The time to reach steady state increases with increasing ρ , c_p and D and with decreasing h.

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