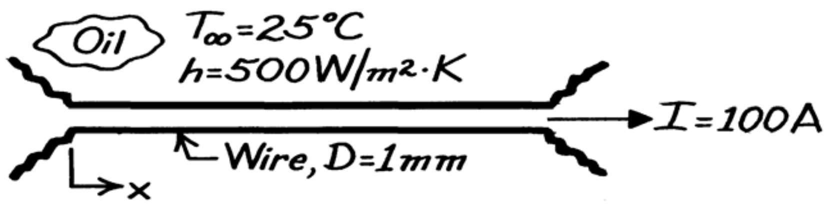
PROBLEM 5.18

KNOWN: Diameter, resistance and current flow for a wire. Convection coefficient and temperature of surrounding oil.

FIND: Steady-state temperature of the wire. Time for the wire temperature to come within 1°C of its steady-state value.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Wire temperature is independent of x.

PROPERTIES: Wire (given): $\rho = 8000\ \text{kg/m}^3$, $c_p = 500\ \text{J/kg·K}$, $k = 20\ \text{W/m·K}$, $R'_e = 0.01\ \Omega/\text{m}$.

ANALYSIS: Since

$$\text{Bi} = \frac{h(r_o/2)}{k} = \frac{500\ \text{W/m}^2\cdot\text{K}\left(2.5\times10^{-4}\text{m}\right)}{20\ \text{W/m}\cdot\text{K}} = 0.006 < 0.1$$

the lumped capacitance method can be used. The problem has been analyzed in Example 1.3, and without radiation the steady-state temperature is given by

$$\pi\ Dh\left(T - T_\infty\right) = I^2 R'_e.$$

Hence

$$T = T_\infty + \frac{I^2 R'_e}{\pi\ Dh} = 25^\circ\text{C} + \frac{(100\text{A})^2\ 0.01\Omega/\text{m}}{\pi(0.001\ \text{m})500\ \text{W/m}^2\cdot\text{K}} = 88.7^\circ\text{C}. \qquad <$$

With no radiation, the transient thermal response of the wire is governed by the expression (Example 1.3)

$$\frac{dT}{dt} = \frac{I^2 R'_e}{\rho c_p\left(\pi D^2/4\right)} - \frac{4h}{\rho\ c_p D}\left(T - T_\infty\right).$$

With $T = T_i = 25°\text{C}$ at $t = 0$, the solution is

$$\frac{T - T_\infty - \left(I^2 R'_e/\pi\ Dh\right)}{T_i - T_\infty - \left(I^2 R'_e/\pi\ Dh\right)} = \exp\left(-\frac{4h}{\rho\ c_p D}t\right).$$

Substituting numerical values, find

$$\frac{87.7 - 25 - 63.7}{25 - 25 - 63.7} = \exp\left(-\frac{4\times500\ \text{W/m}^2\cdot\text{K}}{8000\ \text{kg/m}^3\times500\ \text{J/kg}\cdot\text{K}\times0.001\ \text{m}}t\right)$$

$$t = 8.31\text{s}. \qquad <$$

COMMENTS: The time to reach steady state increases with increasing ρ, c_p and D and with decreasing h.