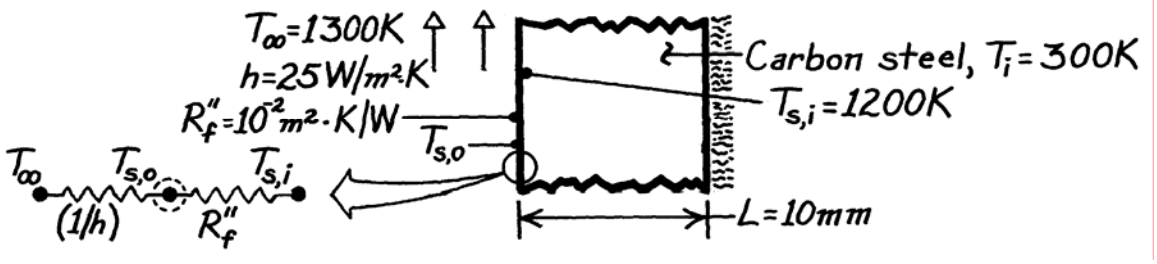
PROBLEM 5.16

KNOWN: Thickness and properties of furnace wall. Thermal resistance of film on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of film surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible film thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel (given): $\rho = 7850$ kg/m^3, $c = 430$ J/kg·K, $k = 60$ W/m·K.

ANALYSIS: The overall coefficient for heat transfer from the surface of the steel to the gas is

$$U = \left(R''_{tot}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\,\text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

Hence,

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K}\times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033$$

and the lumped capacitance method can be used.

(a) It follows that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-t/\tau_t\right) = \exp\left(-t/RC\right) = \exp\left(-Ut/\rho Lc\right)$$

$$t = -\frac{\rho Lc}{U}\ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3\left(0.01\ \text{m}\right)430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1200-1300}{300-1300}$$

$$t = 3886\text{s} = 1.08\text{h}. \quad <$$

(b) Performing an energy balance at the outer surface (s,o),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + \left(1/R''_f\right)} = \frac{25\ \text{W/m}^2\cdot\text{K}\times 1300\ \text{K} + 1200\ \text{K}/10^{-2}\text{m}^2\cdot\text{K/W}}{\left(25+100\right)\text{W/m}^2\cdot\text{K}}$$

$$T_{s,o} = 1220\ \text{K}. \quad <$$

COMMENTS: The film increases τ_t by increasing R_t but not C_t.