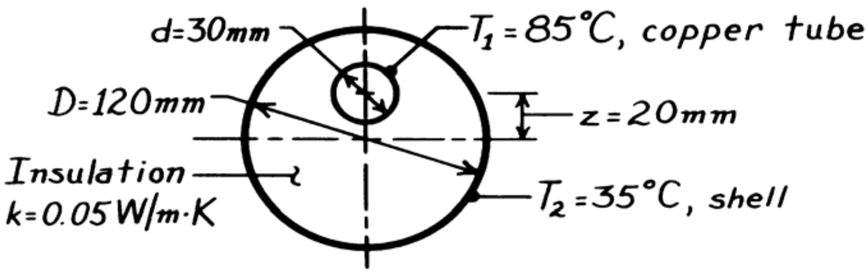
PROBLEM 4.16

KNOWN: Thin-walled copper tube enclosed by an eccentric cylindrical shell; intervening space filled with insulation.

FIND: Heat loss per unit length of tube; compare result with that of a concentric tube-shell arrangement.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Thermal resistances of copper tube wall and outer shell wall are negligible, (4) Two-dimensional conduction in insulation.

ANALYSIS: The heat loss per unit length written in terms of the shape factor S is $q' = k\left(S/\ell\right)\left(T_1 - T_2\right)$ and from Table 4.1 for this geometry,

$$\frac{S}{\ell} = 2\pi / \cosh^{-1}\left[\frac{D^2 + d^2 - 4z^2}{2Dd}\right].$$

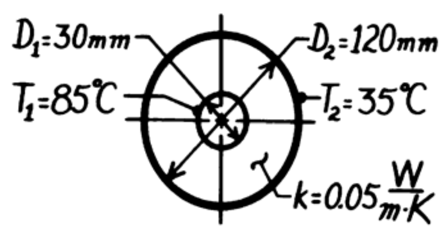
Substituting numerical values, all dimensions in mm,

$$\frac{S}{\ell} = 2\pi / \cosh^{-1}\left[\frac{120^2 + 30^2 - 4\left(20\right)^2}{2 \times 120 \times 30}\right] = 2\pi / \cosh^{-1}\left(1.903\right) = 4.991.$$

Hence, the heat loss is

$$q' = 0.05\,\text{W/m}\cdot\text{K} \times 4.991\left(85 - 35\right)^\circ\text{C} = 12.5\ \text{W/m}. \quad <$$

If the copper tube were concentric with the shell, but all other conditions were the same, the heat loss would be



$$q_c' = \frac{2\pi\ k\left(T_1 - T_2\right)}{\ell n\left(D_2 / D_1\right)}$$

using Eq. 3.27. Substituting numerical values,

$$q_c' = 2\pi \times 0.05\frac{\text{W}}{\text{m}\cdot\text{K}}\left(85 - 35\right)^\circ\text{C}/\ell n\left({}^{120}/_{30}\right)$$

$$q_c' = 11.3\ \text{W/m}.$$

COMMENTS: As expected, the heat loss with the eccentric arrangement is larger than that for the concentric arrangement. The effect of the eccentricity is to increase the heat loss by (12.5 - 11.3)/11.3 ≈ 11%.