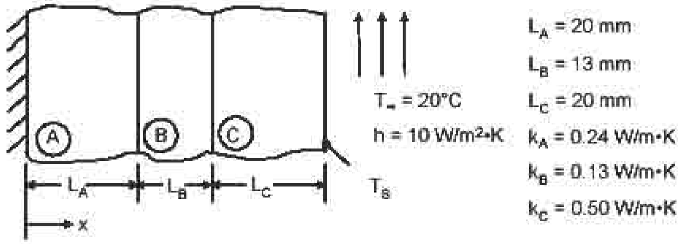
PROBLEM 3.74

KNOWN: Dimensions and properties of a composite wall exposed to convective or insulated conditions.

FIND: (a) Maximum wall temperature for left face insulated and right face convectively cooled. (b) Sketch the steady-state temperature distribution of part (a). (c) Sketch the steady-state temperature distribution with reversed boundary conditions.

SCHEMATIC:



Case 1: $\dot{q}_A$ = 5000 W/m³; $\dot{q}_B = \dot{q}_C = 0$

Case 2: $\dot{q}_A = \dot{q}_B = 0$; $\dot{q}_C$ = 5000 W/m³

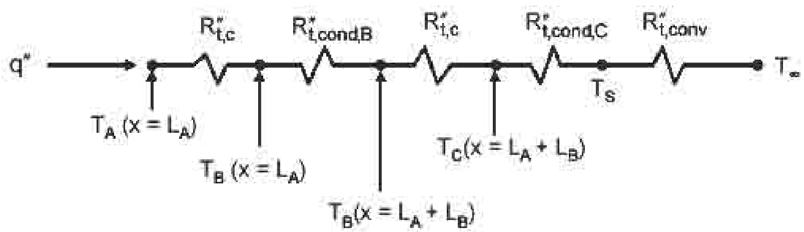
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Uniform volumetric energy generation.

ANALYSIS:

(a) The heat flux through materials B and C is constant and is

$$q'' = \dot{q}_A(L_A) = 5000\ \text{W/m}^3 \times 0.02\ \text{m} = 100\ \text{W/m}^2$$

The thermal resistance network that spans from $x = L_A$ to the coolant is



The thermal resistances are:

$$R''_{t,c} = 0.01\text{m}^2 \cdot \text{K/w}$$

$$R''_{t,cond,B} = \frac{L_B}{k_B} = \frac{0.013\ \text{m}}{0.13\ \text{W/m}\cdot\text{K}} = 0.1\ \frac{\text{m}^2\cdot\text{K}}{\text{W}}$$

$$R''_{t,cond,C} = \frac{L_C}{k_C} = \frac{0.020\ \text{m}}{0.50\ \text{W/m}\cdot\text{K}} = 0.04\ \frac{\text{m}^2\cdot\text{K}}{\text{W}}$$

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$$R''_{t,conv} = \frac{1}{h} = \frac{1}{10\ W/m^2 \cdot K} = 0.1\ \frac{m^2 \cdot K}{W}$$

The total thermal resistance is

$$R''_{t,tot} = (0.01 + 0.1 + 0.01 + 0.04 + 0.1) = 0.26\ \frac{m^2 \cdot K}{W}$$

Therefore,

$$T_A(x{=}L_A) = q''(R''_{t,tot}) + T_\infty = 100\ W/m^2 \times 0.26\ \frac{m^2 \cdot K}{W} + 20°C = 46°C$$

The maximum temperature occurs at x = 0 and may be evaluated by using Eq. 3.43 as follows

$$T_A(x{=}0) = T_A(x{=}L_A) + \frac{\dot{q}_A L_A^2}{2k_A} = 46°C + \frac{5000\ W/m^3 \times (0.02\ m)^2}{2 \times 0.24\ W/m \cdot K}$$

$$T_A(x{=}0) = T_{max} = 50.2°C$$ <

(b) To sketch the temperature distribution, we begin by evaluating the temperatures shown in the thermal resistance network. Working from the coolant side,

$$T_s = T_\infty + q''(R''_{t,conv}) = 20°C + 100\ W/m^2 \times 0.1\ m^2 \cdot K/W = 30°C$$

$$T_C(x = L_A + L_B) = T_s + q''(R''_{t,cond,C}) = 30°C + 100\ W/m^2 \times 0.04\ m^2 \cdot K/W = 34°C$$

$$T_B(x = L_A + L_B) = T_C(x = L_A + L_B) + q''(R''_{t,c}) = 34°C + 100\ W/m^2 \times 0.01\ \frac{m^2 \cdot K}{W} = 35°C$$

$$T_B(x = L_A) = T_B(x = L_A + L_B) + q''(R''_{t,cond,B}) = 35°C + 100\ W/m^2 \times 0.1\ m^2 \cdot K/W = 45°C$$

and from part (a), $T_A(x = L_A) = 46°C$. The temperature distribution is sketched below.

(c) For case 2, the heat flux in the range $0 \le x \le L_A + L_B$ is zero. Hence the boundary at $x = L_A + L_B$ acts as an insulated surface for material C. Therefore, from Eq. 3.43,

$$T_{max} = T_c(x = L_A + L_B) = T_s + \frac{\dot{q} L_C^2}{2k_C} = 30°C + \frac{5000\ W/m^3 \times (0.02m)^2}{2 \times 0.50\ W/m \cdot K} = 32°C$$

The temperature distribution is sketched below.

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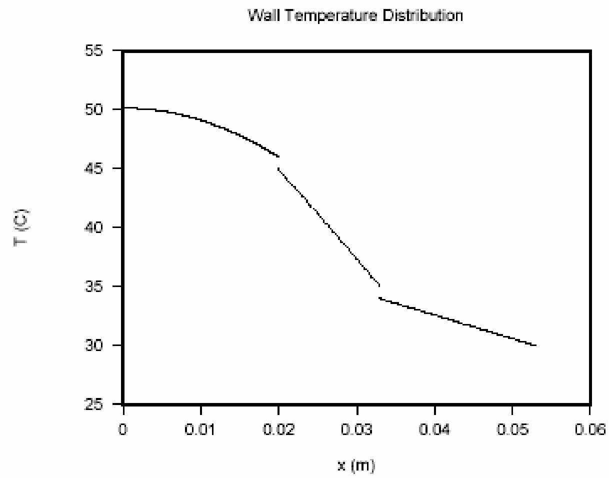
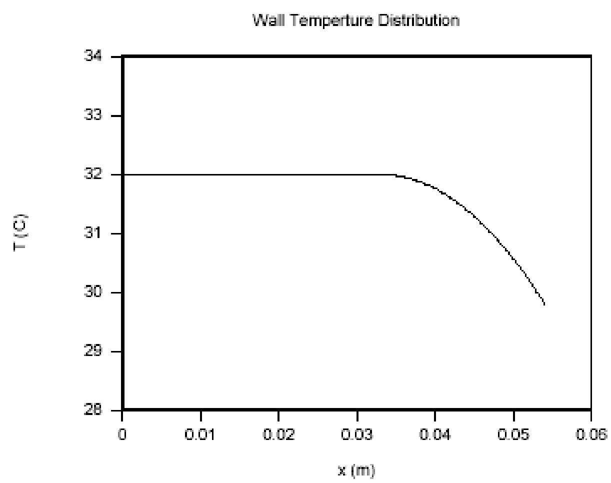
PROBLEM 3.74 (Cont.)



Case 1 temperature distribution.



Case 2 temperature distribution.

COMMENTS: If the heat flux due to conduction in the x-direction is zero, the temperature gradient, dT/dx, must be zero. This is a direct consequence of Fourier's law, and holds under all conditions.