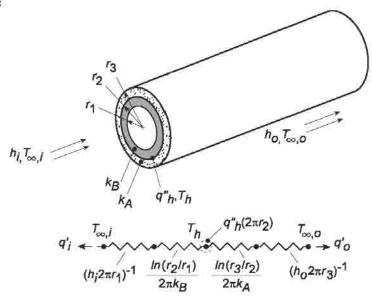
PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\hat{E}_{in} = \hat{E}_{out}$, it follows that

$$q_{h}''(2\pi r_{2}) = q_{i}' + q_{o}' = \frac{T_{h} - T_{\infty,i}}{\left(h_{i} 2\pi r_{l}\right)^{-1} + \frac{\ln\left(r_{2}/r_{l}\right)}{2\pi k_{B}}} + \frac{T_{h} - T_{\infty,o}}{\left(h_{o} 2\pi r_{3}\right)^{-1} + \frac{\ln\left(r_{3}/r_{2}\right)}{2\pi k_{A}}}$$

(c) From the circuit,

$$\frac{q_o'}{q_i'} = \frac{\left(T_h - T_{\infty,o}\right)}{\left(T_h - T_{\infty,i}\right)} \times \frac{\left(h_i 2\pi \eta_i\right)^{-1} + \frac{\ln\left(r_2/\eta_i\right)}{2\pi k_B}}{\left(h_o 2\pi r_3\right)^{-1} + \frac{\ln\left(r_3/r_2\right)}{2\pi k_A}}$$

To reduce q'_0/q'_1 , one could increase k_B , h_i , and r_3/r_2 , while reducing k_A , h_o and r_3/r_1 .

COMMENTS: Contact resistances between the heater and materials A and B could be important.

Excerpts from this work may be reproduced by instructors for distribution on a not-for-profit basis for testing or instructional purposes only to students enrolled in courses for which the textbook has been adopted. Any other reproduction or translation of this work beyond that permitted by Sections 107 or 108 of the 1976 United States Copyright Act without the permission of the copyright owner is unlawful.