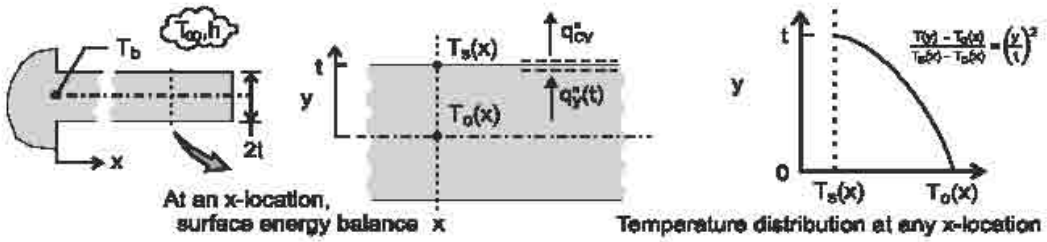
PROBLEM 3.118

KNOWN: Extended surface of rectangular cross-section with heat flow in the longitudinal direction.

FIND: Determine the conditions for which the transverse (y-direction) temperature difference is negligible compared to the temperature difference between the surface and the environment, such that the 1-D analysis of Section 3.6.1 is valid by finding: (a) An expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o, assuming the transverse temperature distribution is parabolic, (b) An expression for the convection heat flux at the surface for the x-location; equate the two expressions, and identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$; and (c) Developing a criterion for the validity of the 1-D assumption used to model an extended surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform convection coefficient and (3) Constant properties.

ANALYSIS: (a) Referring to the schematics above, the conduction heat flux at the surface y = t at any x-location follows from Fourier's law using the parabolic transverse temperature distribution.

$$q''_y(t) = -k\left(\frac{\partial T}{\partial y}\right)_{y=t} = -k\left(\left[T_s(x) - T_o(x)\right]\frac{2y}{t^2}\right)_{y=t} = -\frac{2k}{t}\left[T_s(x) - T_o(x)\right] \tag{1}$$

(b) The convection heat flux at the surface of any x-location follows from the rate equation

$$q''_{cv} = h\left[T_s(x) - T_\infty\right] \tag{2}$$

Performing a surface energy balance as represented schematically above, equating Eqs. (1) and (2) provides

$$q''_y(t) = q''_{cv}$$

$$-\frac{2k}{t}\left[T_s(x) - T_o(x)\right] = h\left[T_s(x) - T_\infty\right]$$

$$\frac{T_s(x) - T_o(x)}{T_s(x) - T_\infty(x)} = -0.5\frac{ht}{k} = -0.5\ Bi \tag{3}$$

where Bi = ht/k, the Biot number, represents the ratio of the conduction to the convection thermal resistances,

$$Bi = \frac{R''_{cd}}{R''_{cv}} = \frac{t/k}{1/h} \tag{4}$$

(c) The transverse temperature difference $(T_s - T_o)$ will be negligible compared to the temperature difference between the surface and the environment $(T_s - T_\infty)$ when Bi << 1, say, 0.1, an order of magnitude smaller. This is the criterion to validate the one-dimensional assumption used to model extended surfaces.

COMMENTS: The coefficient 0.5 in Eq. (3) is a consequence of the parabolic distribution assumption. This distribution represents the simplest polynomial expression that could approximate the real distribution.