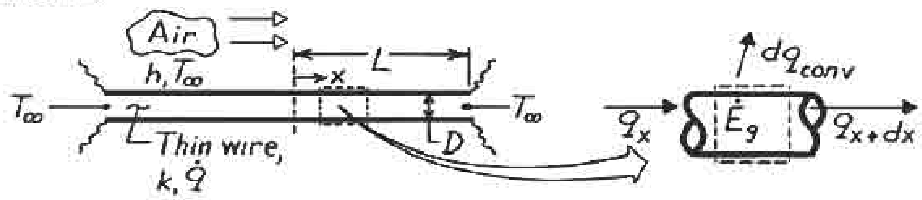
PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi D^2/4\right)dT/dx$$

$$dq_{conv} = h(\pi D\,dx)\ (T - T_\infty) \qquad \dot{E}_g = \dot{q}\left(\pi D^2/4\right)dx.$$

Hence,

$$k\left(\pi D^2/4\right)\frac{d^2T}{dx^2}dx + \dot{q}\left(\pi D^2/4\right)dx - h(\pi D dx)\ (T - T_\infty) = 0$$

or, with $\theta \equiv T - T_\infty$,

$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0 = m C_1 e^0 - mC_2 e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta(L) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh mx}{\cosh mL} - 1\right].$$ <

COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that $T(L) = T(-L) = T_\infty$.