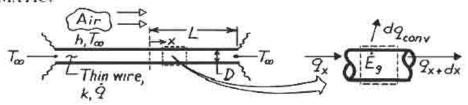
PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire. (3) Constant properties. (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$\begin{split} q_X + \hat{E}_g - dq_{conv} - q_{X+dX} &= 0 \\ \\ q_{X+dX} &= q_X + \frac{dq_X}{dx} dx \qquad q_X = -k \Big(\pi |D^2|/4\Big) dT/dx \\ \\ dq_{conv} &= h \Big(\pi |D| dx\Big) / (T - T_\infty) \qquad \hat{E}_g = \hat{q} \Big(\pi |D^2|/4\Big) dx. \end{split}$$

Hence.

$$k\left(\pi |D^2/4\right) \frac{d^2T}{dx^2} dx + \dot{q}\left(\pi |D^2/4\right) dx - h\left(\pi |Ddx\right) \left(T - T_{\infty}\right) = 0$$
 or, with $\theta = T - T_{\infty}$,
$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1 e^{imx} + C_2 e^{-imx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\frac{d\theta}{dx}\Big|_{x=0} = 0 = m C_1 e^0 - mC_2 e^0 \rightarrow C_1 = C_2$$

$$\theta(L) = 0 = C_1 \left(e^{mL} + e^{-mL} \right) + \frac{\bar{q}}{km^2} \rightarrow C_1 = \frac{-\bar{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_{\infty} - \frac{\dot{q}}{km^2} \left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1 \right] = T_{\infty} - \frac{\dot{q}}{km^2} \left[\frac{\cosh mx}{\cosh mL} - 1 \right]. \tag{$<$}$$

COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that $T(L) = T(-L) = T_{\infty}$.

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