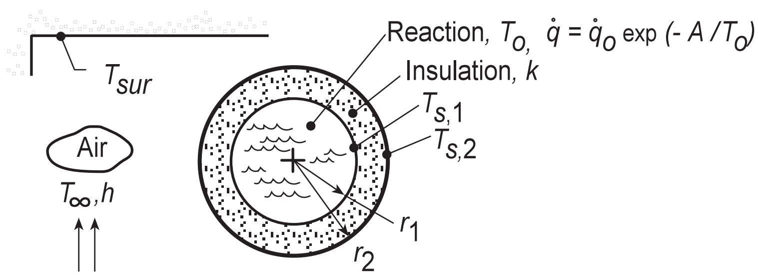
PROBLEM 2.43

KNOWN: Spherical container with an exothermic reaction enclosed by an insulating material whose outer surface experiences convection with adjoining air and radiation exchange with large surroundings.

FIND: (a) Verify that the prescribed temperature distribution for the insulation satisfies the appropriate form of the heat diffusion equation; sketch the temperature distribution and label key features; (b) Applying Fourier's law, verify the conduction heat rate expression for the insulation layer, q_r, in terms of $T_{s,1}$ and $T_{s,2}$; apply a surface energy balance to the container and obtain an alternative expression for q_r in terms of $\dot{q}$ and r_1; (c) Apply a surface energy balance around the outer surface of the insulation to obtain an expression to evaluate $T_{s,2}$; (d) Determine $T_{s,2}$ for the specified geometry and operating conditions; (e) Compute and plot the variation of $T_{s,2}$ as a function of the outer radius for the range $201 \le r_2 \le 210$ mm; explore approaches for reducing $T_{s,2} \le$ 45°C to eliminate potential risk for burn injuries to personnel.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial spherical conduction, (2) Isothermal reaction in container so that $T_o = T_{s,1}$, (2) Negligible thermal contact resistance between the container and insulation, (3) Constant properties in the insulation, (4) Surroundings large compared to the insulated vessel, and (5) Steady-state conditions.

ANALYSIS: The appropriate form of the heat diffusion equation (HDE) for the insulation follows from Eq. 2.27,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0 \qquad (1) <$$

The temperature distribution is given as

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right] \qquad (2)$$

Substitute T(r) into the HDE to see if it is satisfied:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\left[0-\left(T_{s,1}-T_{s,2}\right)\frac{0+\left(r_1/r^2\right)}{1-(r_1/r_2)}\right]\right) = 0$$

$$\frac{1}{r^2}\frac{d}{dr}\left(+\left(T_{s,1}-T_{s,2}\right)\frac{r_1}{1-(r_1/r_2)}\right) = 0 \qquad <$$

and since the expression in parenthesis is independent of r, T(r) does indeed satisfy the HDE. The temperature distribution in the insulation and its key features are as follows:

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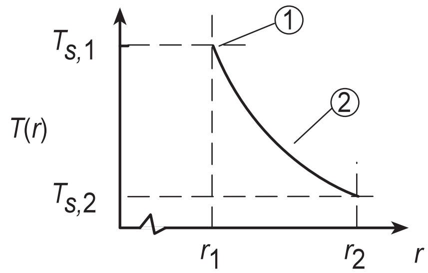
(1) $T_{s,1} > T_{s,2}$

(2) Decreasing gradient with increasing radius, r, since the heat rate is constant through the insulation.



(b) Using Fourier's law for the radial-spherical coordinate, the heat rate through the insulation is

$$q_r = -kA_r \frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$ <

and substituting for the temperature distribution, Eq. (2),

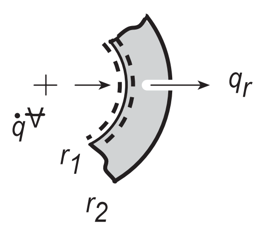
$$q_r = -4k\pi r^2\left[0 - \left(T_{s,1} - T_{s,2}\right)\frac{0 + \left(r_1/r^2\right)}{1 - \left(r_1/r_2\right)}\right]$$

$$q_r = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1\right) - \left(1/r_2\right)} \qquad (3)$$ <

Applying an energy balance to a control surface about the container at $r = r_1$,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\dot{q}\forall - q_r = 0$$



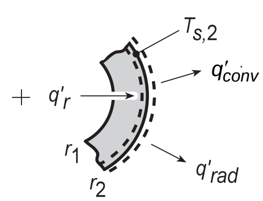
where $\dot{q}\forall$ represents the generated heat in the container,

$$q_r = \left(4/3\right)\pi r_1^3\dot{q} \qquad (4)$$ <

(c) Applying an energy balance to a control surface placed around the outer surface of the insulation,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_r - q_{conv} - q_{rad} = 0$$



$$q_r - hA_s\left(T_{s,2} - T_\infty\right) - \varepsilon A_s\sigma\left(T_{s,2}^4 - T_{sur}^4\right) = 0 \qquad (5)$$ <

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where

$$A_s = 4\pi r_2^2 \tag{6}$$

These relations can be used to determine $T_{s,2}$ in terms of the variables $\dot{q}$, r_1, r_2, h, T_∞, ε and T_{sur}.

(d) Consider the reactor system operating under the following conditions:

r_1 = 200 mm	h = 5 W/m^2·K	ε = 0.9
r_2 = 208 mm	T_∞ = 25°C	T_{sur} = 35°C
k = 0.05 W/m·K		

The heat generated by the exothermic reaction provides for a volumetric heat generation rate,

$$\dot{q} = \dot{q}_o \exp(-A/T_o) \qquad q_o = 5000\,\mathrm{W/m^3} \qquad A = 75\,\mathrm{K} \tag{7}$$

where the temperature of the reaction is that of the inner surface of the insulation, $T_o = T_{s,1}$. The following system of equations will determine the operating conditions for the reactor.

Conduction rate equation, insulation, Eq. (3),

$$q_r = \frac{4\pi \times 0.05\,\mathrm{W/m\cdot K}\left(T_{s,1} - T_{s,2}\right)}{\left(1/0.200\,\mathrm{m} - 1/0.208\,\mathrm{m}\right)} \tag{8}$$

Heat generated in the reactor, Eqs. (4) and (7),

$$q_r = 4/3\,\pi\left(0.200\,\mathrm{m}\right)^3 \dot{q} \tag{9}$$

$$\dot{q} = 5000\,\mathrm{W/m^3} \exp\left(-75\,\mathrm{K}/T_{s,1}\right) \tag{10}$$

Surface energy balance, insulation, Eqs. (5) and (6),

$$q_r - 5\,\mathrm{W/m^2\cdot K}\,A_s\left(T_{s,2} - 298\,\mathrm{K}\right) - 0.9A_s\,5.67\times10^{-8}\,\mathrm{W/m^2{\cdot}K^4}\left(T_{s,2}^4 - \left(308\,\mathrm{K}\right)^4\right) = 0 \tag{11}$$

$$A_s = 4\pi\left(0.208\,\mathrm{m}\right)^2 \tag{12}$$

Solving these equations simultaneously, find that

$$T_{s,1} = 94.3^\circ\mathrm{C} \qquad T_{s,2} = 52.5^\circ\mathrm{C}$$ <

That is, the reactor will be operating at $T_o = T_{s,1}$ = 94.3°C, very close to the desired 95°C operating condition.

(e) Using the above system of equations, Eqs. (8)-(12), we have explored the effects of changes in the convection coefficient, h, and the insulation thermal conductivity, k, as a function of insulation thickness, $t = r_2 - r_1$.

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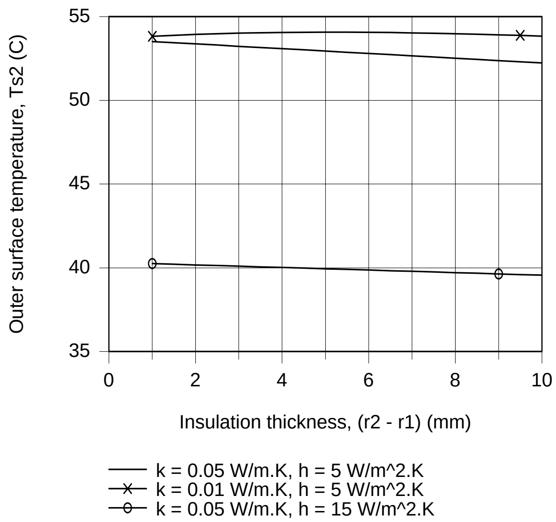
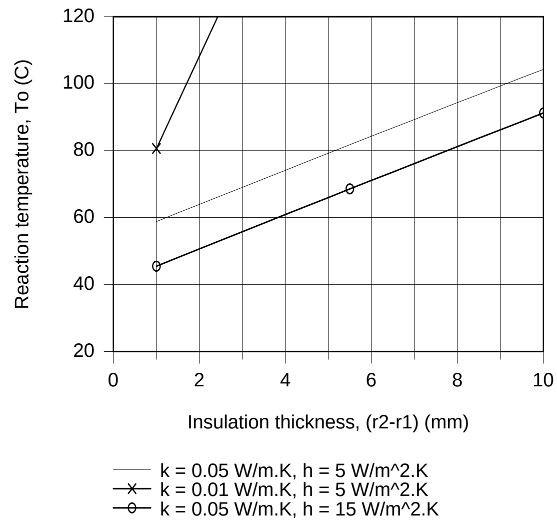
PROBLEM 2.43 (Cont.)





In the $T_{s,2}$ vs. $(r_2 - r_1)$ plot, note that decreasing the thermal conductivity from 0.05 to 0.01 W/m·K slightly increases $T_{s,2}$ while increasing the convection coefficient from 5 to 15 $W/m^2·K$ markedly decreases $T_{s,2}$. Insulation thickness only has a minor effect on $T_{s,2}$ for either option. In the T_o vs. $(r_2 - r_1)$ plot, note that, for all the options, the effect of increased insulation is to increase the reaction temperature. With k = 0.01 W/m·K, the reaction temperature increases beyond 95°C with less than 2 mm insulation. For the case with h = 15 $W/m^2·K$, the reaction temperature begins to approach 95°C with insulation thickness around 10 mm. We conclude that by selecting the proper insulation thickness and controlling the convection coefficient, the reaction could be operated around 95°C such that the outer surface temperature would not exceed 45°C.