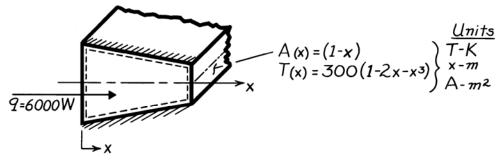
PROBLEM 2.4

KNOWN: Symmetric shape with prescribed variation in cross-sectional area, temperature distribution and heat rate.

FIND: Expression for the thermal conductivity, k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation.

ANALYSIS: Applying the energy balance, Eq. 1.11c, to the system, it follows that, since $\dot{E}_{in} = \dot{E}_{out}$,

 $q_x = Constant \neq f(x).$

Using Fourier's law, Eq. 2.1, with appropriate expressions for A_X and T, yields

$$q_{x} = -k A_{x} \frac{dT}{dx}$$

6000W=-k·(1-x)m²· $\frac{d}{dx} \left[300(1-2x-x^{3}) \right] \frac{K}{m}$

Solving for k and recognizing its units are W/m·K,

$$k = \frac{-6000}{(1-x)\left[300\left(-2-3x^{2}\right)\right]} = \frac{20}{(1-x)\left(2+3x^{2}\right)}.$$

COMMENTS: (1) At x = 0, $k = 10W/m \cdot K$ and $k \to \infty$ as $x \to 1$. (2) Recognize that the 1-D assumption is an approximation which becomes more inappropriate as the area change with x, and hence two-dimensional effects, become more pronounced.

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