PROBLEM 2.28

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q_{x}'' = -k \left[\frac{dT}{dx} \right] = -k \left[-\frac{A}{ka^{2}} (-a) e^{-ax} + B \right] \mathfrak{A}$$

Front Surface, x=0: $q_{x}''(0) = -k \left[+\frac{A}{ka} \cdot 1 + B \right] = -\left[\frac{A}{a} + kB \right]$
Rear Surface, x=L: $q_{x}''(L) = -k \left[+\frac{A}{ka} e^{-aL} + B \right] = -\left[\frac{A}{a} e^{-aL} + kB \right].$

(4)

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$
$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}.$$

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g\,$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}_{g}'' = -\dot{E}_{in}'' + \dot{E}_{out}'' = -q_{x}''(0) + q_{x}''(L) = +\frac{A}{a} \left(1 - e^{-aL}\right).$$

Alternatively, evaluate $\dot{E}_g^{\prime\prime}$ by integration over the volume of the medium,

$$\dot{E}_{g}'' = \int_{0}^{L} \dot{q}(x) dx = \int_{0}^{L} Ae^{-ax} dx = -\frac{A}{a} \left[e^{-ax} \right]_{0}^{L} = \frac{A}{a} \left(1 - e^{-aL} \right).$$

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