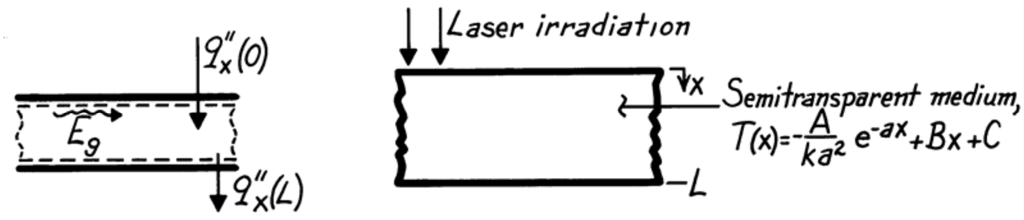
PROBLEM 2.28

KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) Heat generation rate $\dot{q}(x)$, (c) Expression for absorbed radiation per unit surface area in terms of A, a, B, C, L, and k.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]\mathfrak{A}$$

Front Surface, x=0:

$$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right] \qquad <$$

Rear Surface, x=L:

$$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right]. \qquad <$$

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}. \qquad <$$

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

recognize that $\dot{E}_g$ represents the absorbed irradiation. On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right). \qquad <$$

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$