PROBLEM 2.27

KNOWN: Temperature distribution and distribution of heat generation in central layer of a solar pond.

FIND: (a) Heat fluxes at lower and upper surfaces of the central layer, (b) Whether conditions are steady or transient, (c) Rate of thermal energy generation for the entire central layer.

SCHEMATIC:



ASSUMPTIONS: (1) Central layer is stagnant, (2) One-dimensional conduction, (3) Constant properties

ANALYSIS: (a) The desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q_{\text{cond}}'' = -k\frac{\partial T}{\partial x} = -k\left[\frac{A}{ka}e^{-ax} + B\right]$$

Hence,

$$q_{I}'' = q_{cond}'' = -k \left[\frac{A}{ka} e^{-aL} + B \right] \quad q_{u}'' = q_{cond}'' = -k \left[\frac{A}{ka} + B \right].$$

(b) Conditions are steady if $\partial T/\partial t = 0$. Applying the heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \qquad -\frac{A}{k} e^{-ax} + \frac{A}{k} e^{-ax} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Hence conditions are steady since

$$\partial T/\partial t = 0$$
 (for all $0 \le x \le L$).

(c) For the central layer, the energy generation is

$$\begin{split} \dot{E}_{g}'' &= \int_{0}^{L} \dot{q} \, dx = A \, \int_{0}^{L} e^{-ax} \, dx \\ \dot{E}_{g} &= -\frac{A}{a} e^{-ax} \, \left| \begin{array}{c} L \\ 0 \end{array} \right|_{0}^{L} &= -\frac{A}{a} \Big(e^{-aL} - 1 \Big) = \frac{A}{a} \Big(1 - e^{-aL} \Big). \end{split}$$

Alternatively, from an overall energy balance,

$$\begin{aligned} \mathbf{q}_2'' - \mathbf{q}_1'' + \dot{\mathbf{E}}_g'' &= 0 \qquad \dot{\mathbf{E}}_g'' = \mathbf{q}_1'' - \mathbf{q}_2'' = \left(-\mathbf{q}_{\text{cond}(\mathbf{x}=0)}''\right) - \left(-\mathbf{q}_{\text{cond}(\mathbf{x}=L)}''\right) \\ \dot{\mathbf{E}}_g &= \mathbf{k} \left[\frac{\mathbf{A}}{\mathbf{k}\mathbf{a}} + \mathbf{B}\right] - \mathbf{k} \left[\frac{\mathbf{A}}{\mathbf{k}\mathbf{a}} \mathbf{e}^{-\mathbf{a}\mathbf{L}} + \mathbf{B}\right] = \frac{\mathbf{A}}{\mathbf{a}} \left(1 - \mathbf{e}^{-\mathbf{a}\mathbf{L}}\right). \end{aligned}$$

COMMENTS: Conduction is in the negative x-direction, necessitating use of minus signs in the above energy balance.

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