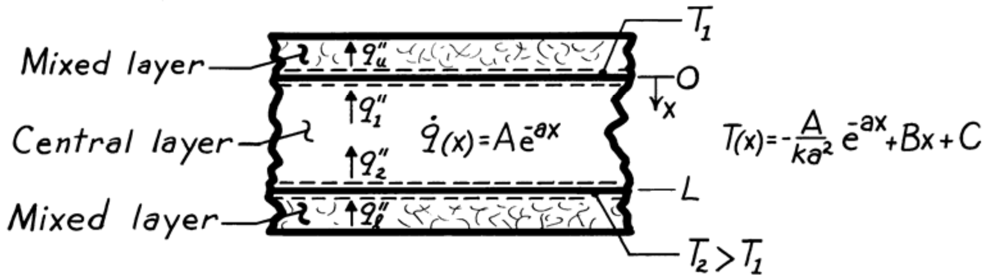
PROBLEM 2.27

KNOWN: Temperature distribution and distribution of heat generation in central layer of a solar pond.

FIND: (a) Heat fluxes at lower and upper surfaces of the central layer, (b) Whether conditions are steady or transient, (c) Rate of thermal energy generation for the entire central layer.

SCHEMATIC:



ASSUMPTIONS: (1) Central layer is stagnant, (2) One-dimensional conduction, (3) Constant properties

ANALYSIS: (a) The desired fluxes correspond to conduction fluxes in the central layer at the lower and upper surfaces. A general form for the conduction flux is

$$q''_{cond} = -k\frac{\partial T}{\partial x} = -k\left[\frac{A}{ka}e^{-ax} + B\right].$$

Hence,

$$q''_l = q''_{cond(x=L)} = -k\left[\frac{A}{ka}e^{-aL} + B\right] \quad q''_u = q''_{cond(x=0)} = -k\left[\frac{A}{ka} + B\right]. \qquad <$$

(b) Conditions are steady if $\partial T/\partial t = 0$. Applying the heat equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad -\frac{A}{k}e^{-ax} + \frac{A}{k}e^{-ax} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Hence conditions are *steady* since

$$\partial T/\partial t = 0 \qquad (\text{for all } 0 \le \times \le L). \qquad <$$

(c) For the central layer, the energy generation is

$$\dot{E}''_g = \int_0^L \dot{q}\,dx = A\int_0^L e^{-ax}\,dx$$

$$\dot{E}_g = -\frac{A}{a}e^{-ax}\Big|_0^L = -\frac{A}{a}\left(e^{-aL} - 1\right) = \frac{A}{a}\left(1 - e^{-aL}\right). \qquad <$$

Alternatively, from an overall energy balance,

$$q''_2 - q''_1 + \dot{E}''_g = 0 \qquad \dot{E}''_g = q''_1 - q''_2 = \left(-q''_{cond(x=0)}\right) - \left(-q''_{cond(x=L)}\right)$$

$$\dot{E}_g = k\left[\frac{A}{ka} + B\right] - k\left[\frac{A}{ka}e^{-aL} + B\right] = \frac{A}{a}\left(1 - e^{-aL}\right).$$

COMMENTS: Conduction is in the negative x-direction, necessitating use of minus signs in the above energy balance.