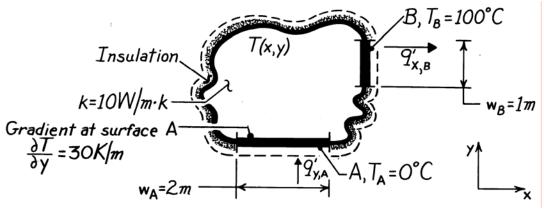
## PROBLEM 2.11

**KNOWN:** Two-dimensional body with specified thermal conductivity and two isothermal surfaces of prescribed temperatures; one surface, A, has a prescribed temperature gradient.

**FIND:** Temperature gradients,  $\partial T/\partial x$  and  $\partial T/\partial y$ , at the surface B.

## **SCHEMATIC:**



**ASSUMPTIONS:** (1) Two-dimensional conduction, (2) Steady-state conditions, (3) No heat generation, (4) Constant properties.

**ANALYSIS:** At the surface A, the temperature gradient in the x-direction must be zero. That is,  $(\partial T/\partial x)_A = 0$ . This follows from the requirement that the heat flux vector must be normal to an isothermal surface. The heat rate at the surface A is given by Fourier's law written as

$$\mathbf{q}_{\mathbf{y},\mathbf{A}}^{\prime} = -\mathbf{k} \cdot \mathbf{w}_{\mathbf{A}} \frac{\partial \mathbf{T}}{\partial \mathbf{y}} \bigg|_{\mathbf{A}} = -10 \frac{\mathbf{W}}{\mathbf{m} \cdot \mathbf{K}} \times 2\mathbf{m} \times 30 \frac{\mathbf{K}}{\mathbf{m}} = -600 \, \text{W/m}.$$

On the surface B, it follows that

$$\left(\partial T/\partial y\right)_{B} = 0$$
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in order to satisfy the requirement that the heat flux vector be normal to the isothermal surface B. Using the conservation of energy requirement, Eq. 1.11c, on the body, find

$$q'_{y,A} - q'_{x,B} = 0$$
 or  $q'_{x,B} = q'_{y,A}$ .

Note that,

$$\mathbf{q}'_{\mathbf{x},\mathbf{B}} = -\mathbf{k} \cdot \mathbf{w}_{\mathbf{B}} \frac{\partial \mathbf{T}}{\partial \mathbf{x}} \bigg]_{\mathbf{B}}$$

and hence

$$\left(\partial T/\partial x\right)_{B} = \frac{-q'_{y,A}}{k \cdot w_{B}} = \frac{-(-600 \text{ W/m})}{10 \text{ W/m} \cdot \text{K} \times 1\text{m}} = 60 \text{ K/m}.$$

**COMMENTS:** Note that, in using the conservation requirement,  $q'_{in} = +q'_{y,A}$  and  $q'_{out} = +q'_{x,B}$ .

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