

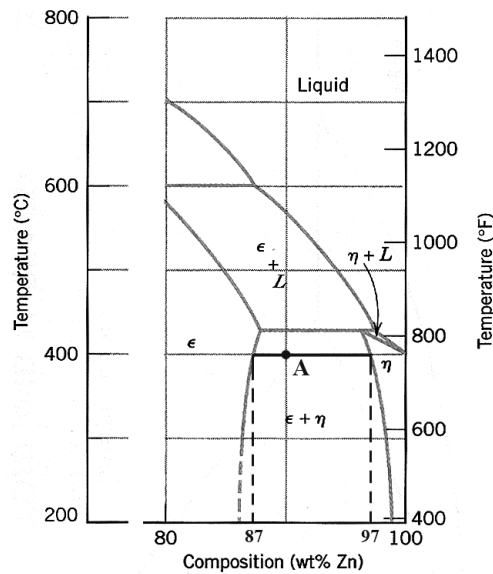
**MAE 20  
Winter 2011  
Assignment 7 solutions**

**9.8 Cite the phases that are present and the phase compositions for the following alloys:**

- (a) 90 wt% Zn-10 wt% Cu at 400°C (750°F)**
- (b) 75 wt% Sn-25 wt% Pb at 175°C (345°F)**
- (c) 55 wt% Ag-45 wt% Cu at 900°C (1650°F)**
- (d) 30 wt% Pb-70 wt% Mg at 425°C (795°F)**
- (e) 2.12 kg Zn and 1.88 kg Cu at 500°C (930°F)**
- (f) 37 lb<sub>m</sub> Pb and 6.5 lb<sub>m</sub> Mg at 400°C (750°F)**
- (g) 8.2 mol Ni and 4.3 mol Cu at 1250°C (2280°F)**
- (h) 4.5 mol Sn and 0.45 mol Pb at 200°C (390°F)**

This problem asks that we cite the phase or phases present for several alloys at specified temperatures.

(a) That portion of the Cu-Zn phase diagram (Figure 9.19) that pertains to this problem is shown below; the point labeled “A” represents the 90 wt% Zn-10 wt% Cu composition at 400°C.

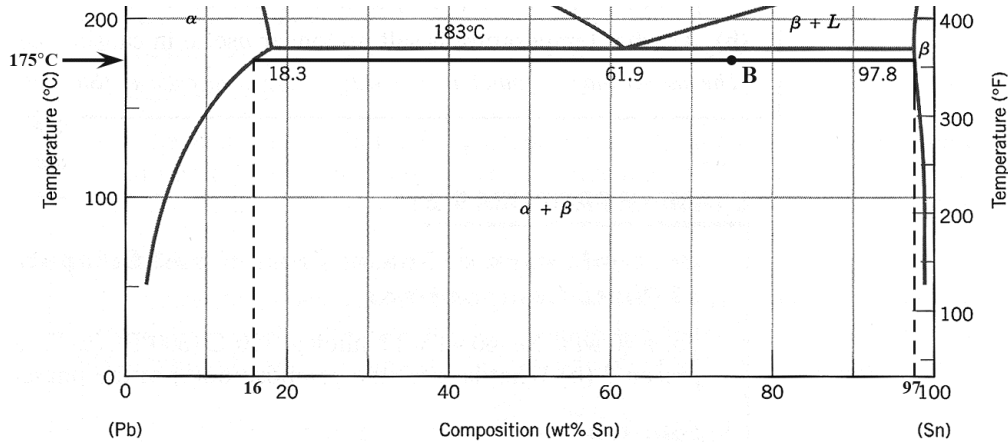


As may be noted, point A lies within the  $\epsilon$  and  $\eta$  phase field. A tie line has been constructed at 400°C; its intersection with the  $\epsilon$ - $\epsilon + \eta$  phase boundary is at 87 wt% Zn, which corresponds to the composition of the  $\epsilon$  phase. Similarly, the tie-line intersection with the  $\epsilon + \eta$ - $\eta$  phase boundary occurs at 97 wt% Zn, which is the composition of the  $\eta$  phase. Thus, the phase compositions are as follows:

$$C_{\epsilon} = 87 \text{ wt\% Zn-13 wt\% Cu}$$

$$C_{\eta} = 97 \text{ wt\% Zn-3 wt\% Cu}$$

(b) That portion of the Pb-Sn phase diagram (Figure 9.8) that pertains to this problem is shown below; the point labeled “B” represents the 75 wt% Sn-25 wt% Pb composition at 175°C.

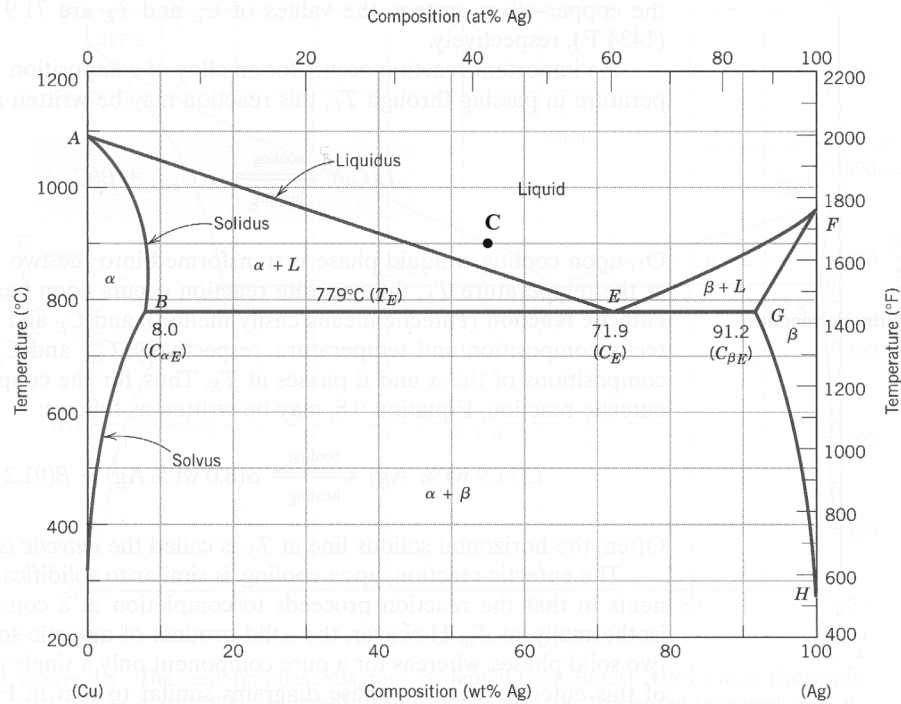


As may be noted, point B lies within the  $\alpha + \beta$  phase field. A tie line has been constructed at 175°C; its intersection with the  $\alpha$ - $\alpha + \beta$  phase boundary is at 16 wt% Sn, which corresponds to the composition of the  $\alpha$  phase. Similarly, the tie-line intersection with the  $\alpha + \beta$ - $\beta$  phase boundary occurs at 97 wt% Sn, which is the composition of the  $\beta$  phase. Thus, the phase compositions are as follows:

$$C_{\alpha} = 16 \text{ wt\% Sn-84 wt\% Pb}$$

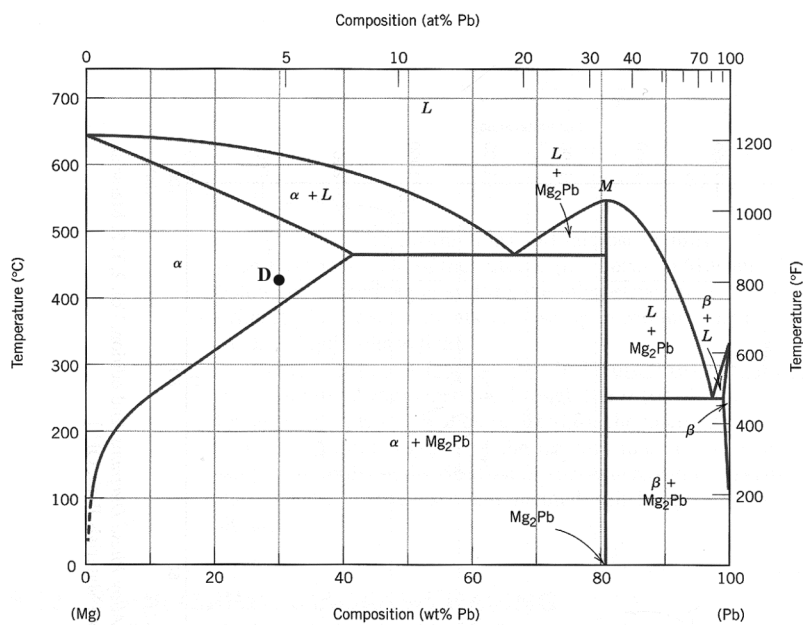
$$C_{\beta} = 97 \text{ wt\% Sn-3 wt\% Pb}$$

(c) The Ag-Cu phase diagram (Figure 9.7) is shown below; the point labeled “C” represents the 55 wt% Ag-45 wt% Cu composition at 900°C.



As may be noted, point C lies within the Liquid phase field. Therefore, only the liquid phase is present; its composition is 55 wt% Ag-45 wt% Cu.

(d) The Mg-Pb phase diagram (Figure 9.20) is shown below; the point labeled “D” represents the 30 wt% Pb-70 wt% Mg composition at 425°C.



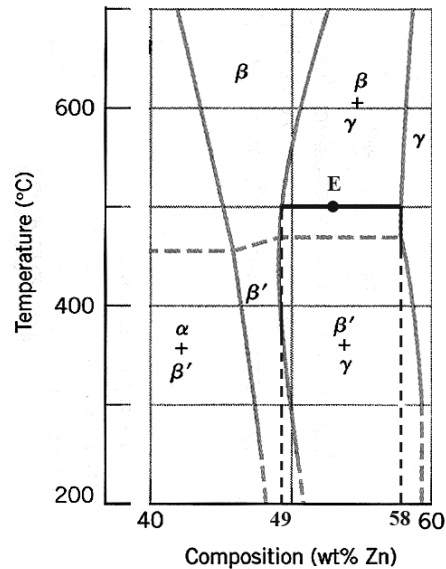
As may be noted, point D lies within the  $\alpha$  phase field. Therefore, only the  $\alpha$  phase is present; its composition is 30 wt% Pb-70 wt% Mg.

(e) For an alloy composed of 2.12 kg Zn and 1.88 kg Cu and at 500°C, we must first determine the Zn and Cu concentrations, as

$$C_{\text{Zn}} = \frac{2.12 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 53 \text{ wt\%}$$

$$C_{\text{Cu}} = \frac{1.88 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 47 \text{ wt\%}$$

That portion of the Cu-Zn phase diagram (Figure 9.19) that pertains to this problem is shown below; the point labeled “E” represents the 53 wt% Zn-47 wt% Cu composition at 500°C.



As may be noted, point E lies within the  $\beta + \gamma$  phase field. A tie line has been constructed at 500°C; its intersection with the  $\beta$ - $\beta + \gamma$  phase boundary is at 49 wt% Zn, which corresponds to the composition of the  $\beta$  phase. Similarly, the tie-line intersection with the  $\beta + \gamma$ - $\gamma$  phase boundary occurs at 58 wt% Zn, which is the composition of the  $\gamma$  phase. Thus, the phase compositions are as follows:

$$C_{\beta} = 49 \text{ wt\% Zn-51 wt\% Cu}$$

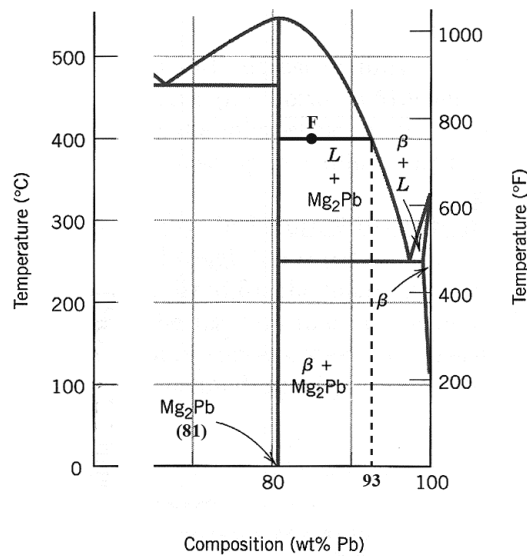
$$C_{\gamma} = 58 \text{ wt\% Zn-42 wt\% Cu}$$

(f) For an alloy composed of 37 lb<sub>m</sub> Pb and 6.5 lb<sub>m</sub> Mg and at 400°C, we must first determine the Pb and Mg concentrations, as

$$C_{\text{Pb}} = \frac{37 \text{ lb}_m}{37 \text{ lb}_m + 6.5 \text{ lb}_m} \times 100 = 85 \text{ wt}\%$$

$$C_{\text{Mg}} = \frac{6.5 \text{ lb}_m}{37 \text{ lb}_m + 6.5 \text{ lb}_m} \times 100 = 15 \text{ wt}\%$$

That portion of the Mg-Pb phase diagram (Figure 9.20) that pertains to this problem is shown below; the point labeled “F” represents the 85 wt% Pb-15 wt% Mg composition at 400°C.



As may be noted, point F lies within the  $L + \text{Mg}_2\text{Pb}$  phase field. A tie line has been constructed at 400°C; it intersects the vertical line at 81 wt% Pb, which corresponds to the composition of  $\text{Mg}_2\text{Pb}$ . Furthermore, the tie line intersection with the  $L + \text{Mg}_2\text{Pb}$ - $L$  phase boundary is at 93 wt% Pb, which is the composition of the liquid phase.

Thus, the phase compositions are as follows:

$$C_{\text{Mg}_2\text{Pb}} = 81 \text{ wt}\% \text{ Pb}-19 \text{ wt}\% \text{ Mg}$$

$$C_L = 93 \text{ wt}\% \text{ Pb}-7 \text{ wt}\% \text{ Mg}$$

(g) For an alloy composed of 8.2 mol Ni and 4.3 mol Cu and at 1250°C, it is first necessary to determine the Ni and Cu concentrations, which we will do in wt% as follows:

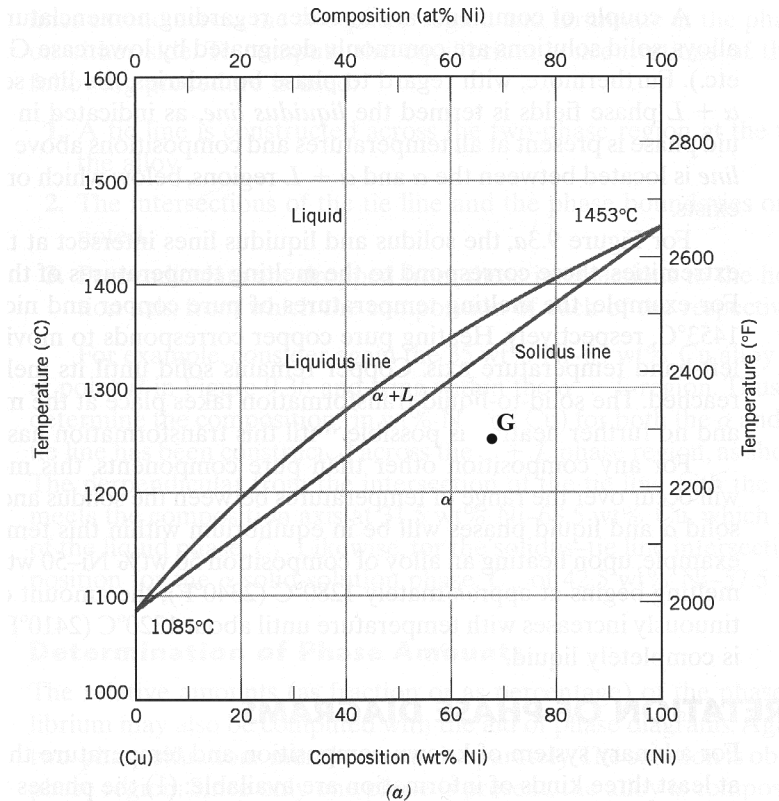
$$n'_{\text{Ni}} = n_{m_{\text{Ni}}} A_{\text{Ni}} = (8.2 \text{ mol})(58.69 \text{ g/mol}) = 481.3 \text{ g}$$

$$n'_{\text{Cu}} = n_{m_{\text{Cu}}} A_{\text{Cu}} = (4.3 \text{ mol})(63.55 \text{ g/mol}) = 273.3 \text{ g}$$

$$C_{\text{Ni}} = \frac{481.3 \text{ g}}{481.3 \text{ g} + 273.3 \text{ g}} \times 100 = 63.8 \text{ wt\%}$$

$$C_{\text{Cu}} = \frac{273.3 \text{ g}}{481.3 \text{ g} + 273.3 \text{ g}} \times 100 = 36.2 \text{ wt\%}$$

The Cu-Ni phase diagram (Figure 9.3a) is shown below; the point labeled “G” represents the 63.8 wt% Ni-36.2 wt% Cu composition at 1250°C.



As may be noted, point G lies within the  $\alpha$  phase field. Therefore, only the  $\alpha$  phase is present; its composition is 63.8 wt% Ni-36.2 wt% Cu.

(h) For an alloy composed of 4.5 mol Sn and 0.45 mol Pb and at 200°C, it is first necessary to determine the Sn and Pb concentrations, which we will do in weight percent as follows:

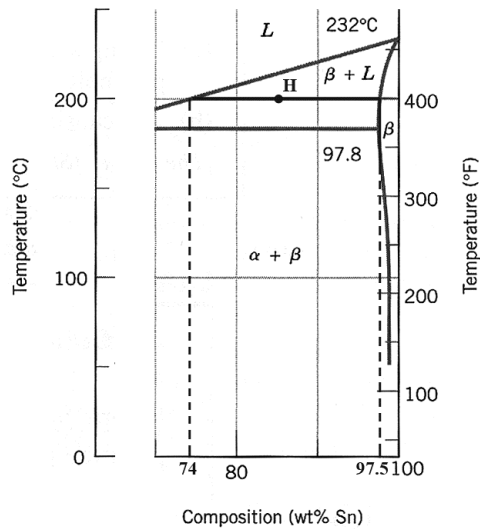
$$n'_{\text{Sn}} = n_{m_{\text{Sn}}} A_{\text{Sn}} = (4.5 \text{ mol})(118.71 \text{ g/mol}) = 534.2 \text{ g}$$

$$n'_{\text{Pb}} = n_{m_{\text{Pb}}} A_{\text{Pb}} = (0.45 \text{ mol})(207.2 \text{ g/mol}) = 93.2 \text{ g}$$

$$C_{\text{Sn}} = \frac{534.2 \text{ g}}{534.2 \text{ g} + 93.2 \text{ g}} \times 100 = 85.1 \text{ wt\%}$$

$$C_{\text{Pb}} = \frac{93.2 \text{ g}}{534.2 \text{ g} + 93.2 \text{ g}} \times 100 = 14.9 \text{ wt\%}$$

That portion of the Pb-Sn phase diagram (Figure 9.8) that pertains to this problem is shown below; the point labeled “H” represents the 85.1 wt% Sn-14.9 wt% Pb composition at 200°C.



As may be noted, point H lies within the  $\beta + L$  phase field. A tie line has been constructed at 200°C; its intersection with the  $L-\beta + L$  phase boundary is at 74 wt% Sn, which corresponds to the composition of the  $L$  phase. Similarly, the tie-line intersection with the  $\beta + L-\beta$  phase boundary occurs at 97.5 wt% Sn, which is the composition of the  $\beta$  phase. Thus, the phase compositions are as follows:

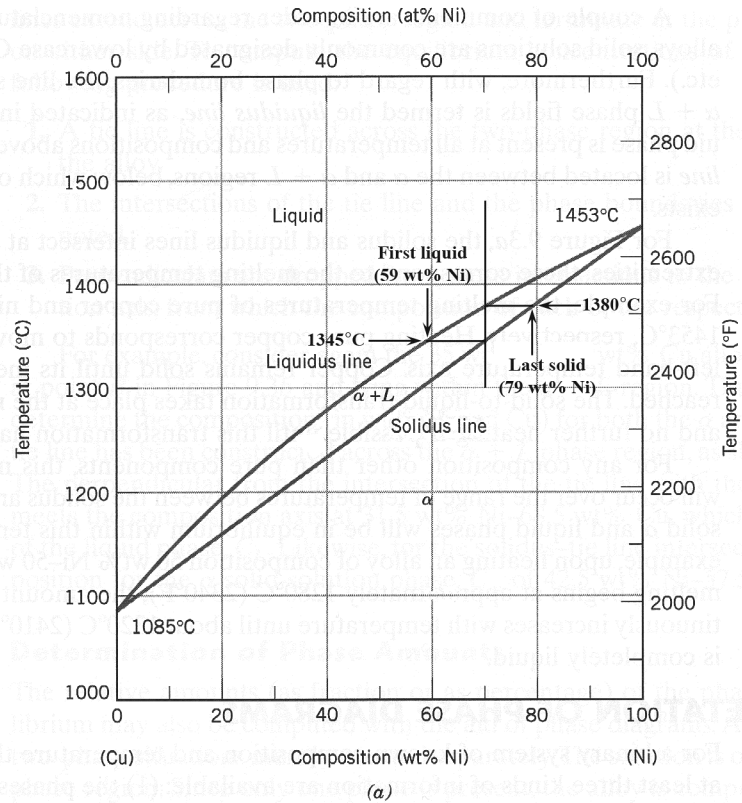
$$C_{\beta} = 97.5 \text{ wt\% Sn}-2.5 \text{ wt\% Pb}$$

$$C_L = 74 \text{ wt\% Sn}-26 \text{ wt\% Pb}$$

**9.11** *A copper-nickel alloy of composition 70 wt% Ni-30 wt% Cu is slowly heated from a temperature of 1300°C (2370°F).*

- (a) At what temperature does the first liquid phase form?*
- (b) What is the composition of this liquid phase?*
- (c) At what temperature does complete melting of the alloy occur?*
- (d) What is the composition of the last solid remaining prior to complete melting?*

Shown below is the Cu-Ni phase diagram (Figure 9.3a) and a vertical line constructed at a composition of 70 wt% Ni-30 wt% Cu.



- (a) Upon heating from 1300°C, the first liquid phase forms at the temperature at which this vertical line intersects the  $\alpha$ -( $\alpha + L$ ) phase boundary--i.e., about 1345°C.
- (b) The composition of this liquid phase corresponds to the intersection with the ( $\alpha + L$ )-L phase boundary, of a tie line constructed across the  $\alpha + L$  phase region at 1345°C--i.e., 59 wt% Ni;
- (c) Complete melting of the alloy occurs at the intersection of this same vertical line at 70 wt% Ni with the ( $\alpha + L$ )-L phase boundary--i.e., about 1380°C;
- (d) The composition of the last solid remaining prior to complete melting corresponds to the intersection with  $\alpha$ -( $\alpha + L$ ) phase boundary, of the tie line constructed across the  $\alpha + L$  phase region at 1380°C--i.e., about 79 wt% Ni.

**9.15** A 1.5-kg specimen of a 90 wt% Pb–10 wt% Sn alloy is heated to 250°C (480°F); at this temperature it is entirely an  $\alpha$ -phase solid solution (Figure 9.8). The alloy is to be melted to the extent that 50% of the specimen is liquid, the remainder being the  $\alpha$  phase. This may be accomplished either by heating the alloy or changing its composition while holding the temperature constant.

- (a) To what temperature must the specimen be heated?
- (b) How much tin must be added to the 1.5-kg specimen at 250°C to achieve this state?



(a) Probably the easiest way to solve this part of the problem is by trial and error--that is, on the Pb-Sn phase diagram (Figure 9.8), moving vertically at the given composition, through the  $\alpha + L$  region until the tie-line lengths on both sides of the given composition are the same. This occurs at approximately 295°C (560°F).

(b) We can also produce a 50% liquid solution at 250°C, by adding Sn to the alloy. At 250°C and within the  $\alpha + L$  phase region

$$C_{\alpha} = 14 \text{ wt\% Sn-86 wt\% Pb}$$

$$C_L = 34 \text{ wt\% Sn-66 wt\% Pb}$$

Let  $C_0$  be the new alloy composition to give  $W_{\alpha} = W_L = 0.5$ . Then,

$$W_{\alpha} = 0.5 = \frac{C_L - C_0}{C_L - C_{\alpha}} = \frac{34 - C_0}{34 - 14}$$

And solving for  $C_0$  gives 24 wt% Sn. Now, let  $m_{\text{Sn}}$  be the mass of Sn added to the alloy to achieve this new composition. The amount of Sn in the original alloy is

$$(0.10)(1.5 \text{ kg}) = 0.15 \text{ kg}$$

Then, using a modified form of Equation 4.3

$$\left[ \frac{0.15 \text{ kg} + m_{\text{Sn}}}{1.5 \text{ kg} + m_{\text{Sn}}} \right] \times 100 = 24$$

And, solving for  $m_{\text{Sn}}$  (the mass of tin to be added), yields  $m_{\text{Sn}} = 0.276 \text{ kg}$ .

**9.19** For alloys of two hypothetical metals A and B, there exist an  $\alpha$ , A-rich phase and a  $\beta$ , B-rich phase. From the mass fractions of both phases for two different alloys provided in the table below, (which are at the same temperature), determine the composition of the phase boundary (or solubility limit) for both  $\alpha$  and  $\beta$  phases at this temperature.

<i>Alloy Composition</i>	<i>Fraction <math>\alpha</math> Phase</i>	<i>Fraction <math>\beta</math> Phase</i>
60 wt% A–40 wt% B	0.57	0.43
30 wt% A–70 wt% B	0.14	0.86

The problem is to solve for compositions at the phase boundaries for both  $\alpha$  and  $\beta$  phases (i.e.,  $C_\alpha$  and  $C_\beta$ ). We may set up two independent lever rule expressions, one for each composition, in terms of  $C_\alpha$  and  $C_\beta$  as follows:

$$W_{\alpha 1} = 0.57 = \frac{C_\beta - C_{01}}{C_\beta - C_\alpha} = \frac{C_\beta - 60}{C_\beta - C_\alpha}$$

$$W_{\alpha 2} = 0.14 = \frac{C_\beta - C_{02}}{C_\beta - C_\alpha} = \frac{C_\beta - 30}{C_\beta - C_\alpha}$$

In these expressions, compositions are given in wt% of A. Solving for  $C_\alpha$  and  $C_\beta$  from these equations, yield

$$C_\alpha = 90 \text{ (or 90 wt\% A-10 wt\% B)}$$

$$C_\beta = 20.2 \text{ (or 20.2 wt\% A-79.8 wt\% B)}$$

**9.50 Consider 1.0 kg of austenite containing 1.15 wt% C, cooled to below 727°C (1341°F).**

- (a) What is the proeutectoid phase?**
- (b) How many kilograms each of total ferrite and cementite form?**
- (c) How many kilograms each of pearlite and the proeutectoid phase form?**
- (d) Schematically sketch and label the resulting microstructure.**

(a) The proeutectoid phase will be  $\text{Fe}_3\text{C}$  since 1.15 wt% C is greater than the eutectoid composition (0.76 wt% C).

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

$$W_\alpha = \frac{C_{\text{Fe}_3\text{C}} - C_0}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{6.70 - 1.15}{6.70 - 0.022} = 0.83$$

which, when multiplied by the total mass of the alloy (1.0 kg), gives 0.83 kg of total ferrite.

Similarly, for total cementite,

$$W_{\text{Fe}_3\text{C}} = \frac{C_0 - C_\alpha}{C_{\text{Fe}_3\text{C}} - C_\alpha} = \frac{1.15 - 0.022}{6.70 - 0.022} = 0.17$$

And the mass of total cementite that forms is  $(0.17)(1.0 \text{ kg}) = 0.17 \text{ kg}$ .

(c) Now we are asked to calculate how much pearlite and the proeutectoid phase (cementite) form. Applying Equation 9.22, in which  $C_1' = 1.15$  wt% C

$$W_p = \frac{6.70 - C_1'}{6.70 - 0.76} = \frac{6.70 - 1.15}{6.70 - 0.76} = 0.93$$

which corresponds to a mass of 0.93 kg. Likewise, from Equation 9.23

$$W_{\text{Fe}_3\text{C}'} = \frac{C_1' - 0.76}{5.94} = \frac{1.15 - 0.76}{5.94} = 0.07$$

which is equivalent to 0.07 kg of the total 1.0 kg mass.

(d) Schematically, the microstructure would appear as:

