MAE 20
Winter 2011
Assignment 5

6.7 *For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).*

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

Solution

(a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 275 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = (275 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)$$

$$= 89{,}375 \text{ N} \quad (20{,}000 \text{ lb}_\text{f})$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (115 \text{ mm})\left[1 + \frac{275 \text{ MPa}}{115 \times 10^3 \text{ MPa}}\right] = 115.28 \text{ mm} \quad (4.51 \text{ in.})$$

6.22 *Consider the brass alloy for which the stress-strain behavior is shown in Figure 6.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lb_f). If it is known that this alloy has a Poisson's ratio of 0.30, compute: (a) the specimen elongation, and (b) the reduction in specimen diameter.*

Solution

(a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation 6.1, as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\dfrac{d_0}{2}\right)^2} = \frac{5000\ \text{N}}{\pi\left(\dfrac{6 \times 10^{-3}\,\text{m}}{2}\right)^2} = 177 \times 10^6\ \text{N/m}^2 = 177\ \text{MPa}\ \ (25{,}000\ \text{psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 2.0×10^{-3}. From the definition of strain, Equation 6.2

$$\Delta l = \varepsilon l_0 = (2.0 \times 10^{-3})(50\ \text{mm}) = 0.10\ \text{mm} \quad (4 \times 10^{-3}\ \text{in.})$$

(b) In order to determine the reduction in diameter Δd, it is necessary to use Equation 6.8 and the definition of lateral strain (i.e., $\varepsilon_x = \Delta d/d_0$) as follows

$$\Delta d = d_0 \varepsilon_x = -d_0 \nu \varepsilon_z = -(6\ \text{mm})(0.30)(2.0 \times 10^{-3})$$

$$= -3.6 \times 10^{-3}\ \text{mm} \ \ (-1.4 \times 10^{-4}\ \text{in.})$$

6.29 *A cylindrical specimen of aluminum having a diameter of 0.505 in. (12.8 mm) and a gauge length of 2.000 in. (50.800 mm) is pulled in tension. Use the load–elongation characteristics tabulated below to complete parts (a) through (f).*

Load		***Length***	
N	***lb_f***	***mm***	***in.***
0	*0*	*50.800*	*2.000*
7,330	*1,650*	*50.851*	*2.002*
15,100	*3,400*	*50.902*	*2.004*
23,100	*5,200*	*50.952*	*2.006*
30,400	*6,850*	*51.003*	*2.008*
34,400	*7,750*	*51.054*	*2.010*
38,400	*8,650*	*51.308*	*2.020*
41,300	*9,300*	*51.816*	*2.040*
44,800	*10,100*	*52.832*	*2.080*

46,200	*10,400*	*53.848*	*2.120*
47,300	*10,650*	*54.864*	*2.160*
47,500	*10,700*	*55.880*	*2.200*
46,100	*10,400*	*56.896*	*2.240*
44,800	*10,100*	*57.658*	*2.270*
42,600	*9,600*	*58.420*	*2.300*
36,400	*8,200*	*59.182*	*2.330*
	Fracture		

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

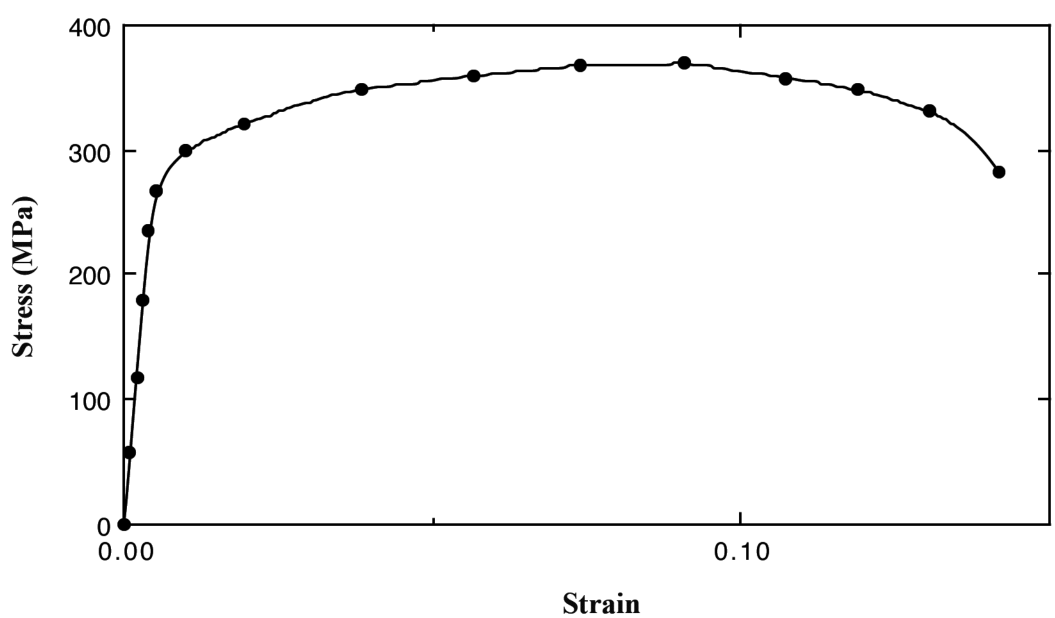
(e) What is the approximate ductility, in percent elongation?

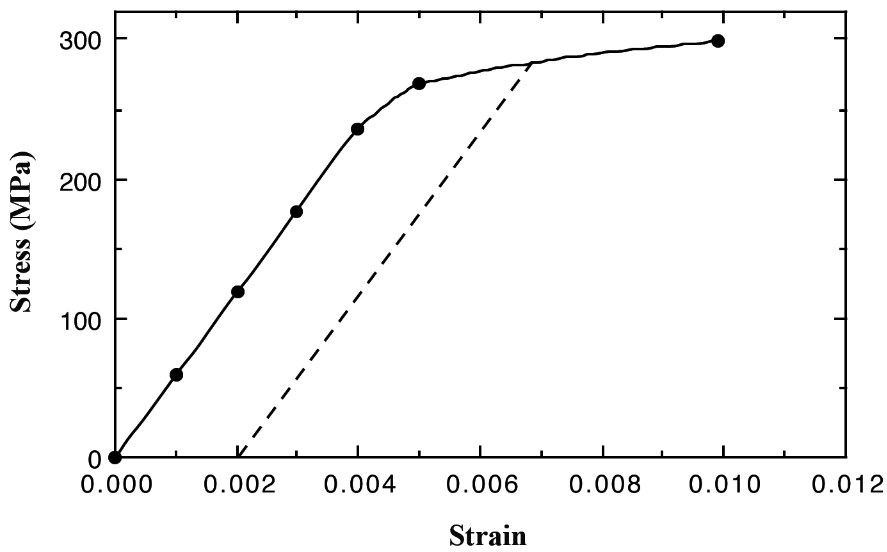
(f) Compute the modulus of resilience.

Solution

This problem calls for us to make a stress-strain plot for aluminum, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends to just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{200 \text{ MPa} - 0 \text{ MPa}}{0.0032 - 0} = 62.5 \times 10^3 \text{ MPa} = 62.5 \text{ GPa} \quad (9.1 \times 10^6 \text{ psi})$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 285 MPa (41,000 psi).

(d) The tensile strength is approximately 370 MPa (54,000 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The total fracture strain at fracture is 0.165; subtracting out the elastic strain (which is about 0.005) leaves a plastic strain of 0.160. Thus, the ductility is about 16%EL.

(f) From Equation 6.14, the modulus of resilience is just

$$U_r = \frac{\sigma_y^2}{2E}$$

which, using data computed above gives a value of

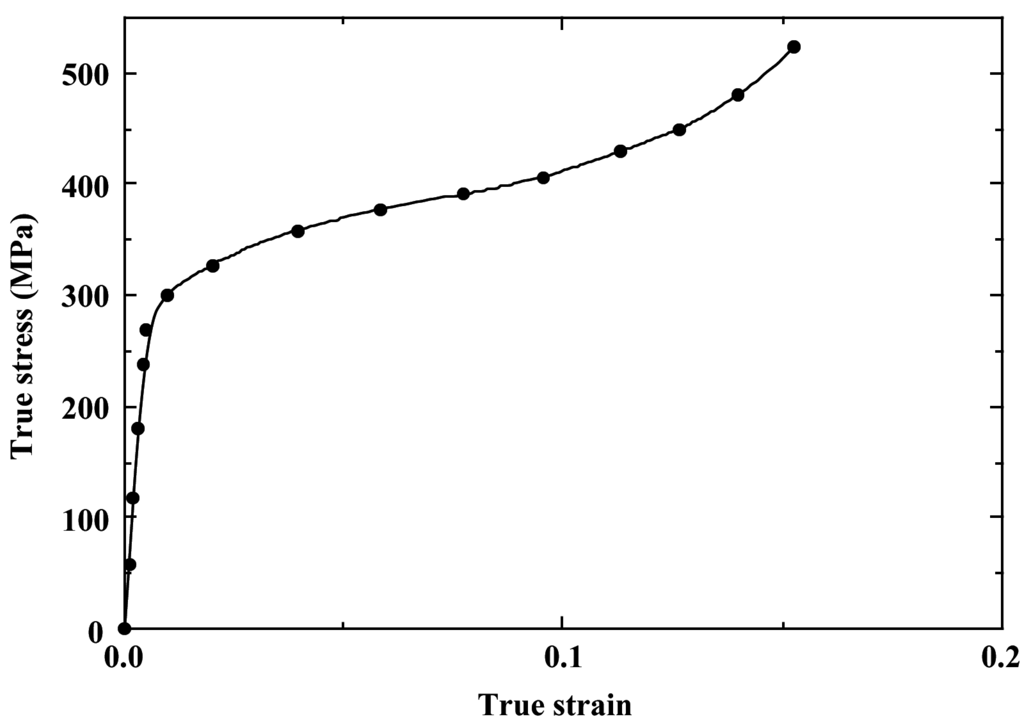
$$U_r = \frac{(285 \text{ MPa})^2}{(2)\left(62.5 \times 10^3 \text{ MPa}\right)} = 0.65 \text{ MN/m}^2 = 0.65 \times 10^6 \text{ N/m}^2 = 6.5 \times 10^5 \text{ J/m}^3 \quad \left(93.8 \text{ in.-lb}_f\text{/in.}^3\right)$$

6.41 *Using the data in Problem 6.28 and Equations 6.15, 6.16, and 6.18a, generate a true stress–true strain plot for aluminum. Equation 6.18a becomes invalid past the point at which necking begins; therefore, measured diameters are given below for the last four data points, which should be used in true stress computations.*

Load		***Length***		***Diameter***	
N	***lb_f***	***mm***	***in.***	***mm***	***in.***
46,100	*10,400*	*56.896*	*2.240*	*11.71*	*0.461*
42,400	*10,100*	*57.658*	*2.270*	*10.95*	*0.431*
42,600	*9,600*	*58.420*	*2.300*	*10.62*	*0.418*
36,400	*8,200*	*59.182*	*2.330*	*9.40*	*0.370*

Solution

These true stress-strain data are plotted below.



6.44 T*he following true stresses produce the corresponding true plastic strains for a brass alloy:*

True Stress (psi)	***True Strain***

50,000	*0.10*
60,000	*0.20*

What true stress is necessary to produce a true plastic strain of 0.25?

Solution

For this problem, we are given two values of ε_T and σ_T, from which we are asked to calculate the true stress which produces a true plastic strain of 0.25. Employing Equation 6.19, we may set up two simultaneous equations with two unknowns (the unknowns being *K* and *n*), as

$$\log\,(50{,}000 \text{ psi}) \;=\; \log K \;+\; n \log\,(0.10)$$

$$\log\,(60{,}000 \text{ psi}) \;=\; \log K \;+\; n \log\,(0.20)$$

Solving for *n* from these two expressions yields

$$n = \frac{\log\,(50{,}000) - \log\,(60{,}000)}{\log\,(0.10) - \log\,(0.20)} \;=\; 0.263$$

and for *K*

$$\log K = 4.96 \text{ or } K = 10^{4.96} = \;91{,}623 \text{ psi}$$

Thus, for $\varepsilon_T = 0.25$

$$\sigma_T \;=\; K\,(\varepsilon_T)^n \;=\; (91{,}623\ \text{psi})(0.25)^{0.263} = 63{,}700\ \text{psi} \quad (440\ \text{MPa})$$