MAE 20
Winter 2011
Assignment 4

7.6 *(a) Compare planar densities (Section 3.11 and Problem 3.54) for the (100), (110), and (111) planes for FCC.*

(b) Compare planar densities (Problem 3.55) for the (100), (110), and (111) planes for BCC.

<u>Solution</u>

(a) For the FCC crystal structure, the planar density for the (110) plane is given in Equation 3.11 as

$$\mathrm{PD}_{110}(\mathrm{FCC}) = \frac{1}{4R^2\sqrt{2}} = \frac{0.177}{R^2}$$

Furthermore, the planar densities of the (100) and (111) planes are calculated in Homework Problem 3.54, which are as follows:

$$\mathrm{PD}_{100}(\mathrm{FCC}) = \frac{1}{4R^2} = \frac{0.25}{R^2}$$

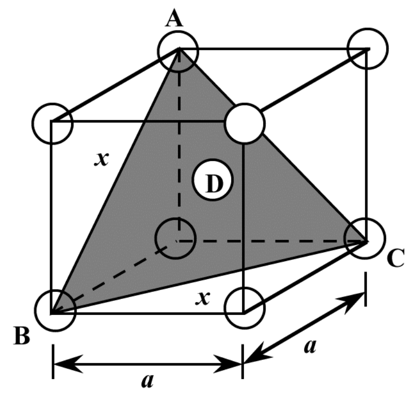
$$\mathrm{PD}_{111}(\mathrm{FCC}) = \frac{1}{2R^2\sqrt{3}} = \frac{0.29}{R^2}$$

(b) For the BCC crystal structure, the planar densities of the (100) and (110) planes were determined in Homework Problem 3.55, which are as follows:

$$\mathrm{PD}_{100}(\mathrm{BCC}) = \frac{3}{16R^2} = \frac{0.19}{R^2}$$

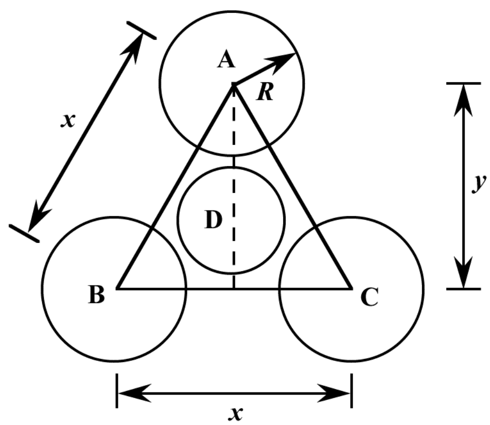
$$\mathrm{PD}_{110}(\mathrm{BCC}) = \frac{3}{8R^2\sqrt{2}} = \frac{0.27}{R^2}$$

Below is a BCC unit cell, within which is shown a (111) plane.



(a)

The centers of the three corner atoms, denoted by A, B, and C lie on this plane. Furthermore, the (111) plane does not pass through the center of atom D, which is located at the unit cell center. The atomic packing of this plane is presented in the following figure; the corresponding atom positions from the Figure (a) are also noted.



(b)

Inasmuch as this plane does not pass through the center of atom D, it is not included in the atom count. One sixth of each of the three atoms labeled A, B, and C is associated with this plane, which gives an equivalence of one-half atom.

In Figure (b) the triangle with A, B, and C at its corners is an equilateral triangle. And, from Figure (b), the area of this triangle is $\frac{xy}{2}$. The triangle edge length, x, is equal to the length of a face diagonal, as indicated in Figure (a). And its length is related to the unit cell edge length, a, as

$$x^2 = a^2 + a^2 = 2a^2$$

or

$$x = a\sqrt{2}$$

For BCC, $a = \frac{4R}{\sqrt{3}}$ (Equation 3.3), and, therefore,

$$x = \frac{4R\sqrt{2}}{\sqrt{3}}$$

Also, from Figure (*b*), with respect to the length *y* we may write

$$y^2 + \left(\frac{x}{2}\right)^2 = x^2$$

which leads to $y = \frac{x\sqrt{3}}{2}$. And, substitution for the above expression for *x* yields

$$y = \frac{x\sqrt{3}}{2} = \left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{4R\sqrt{2}}{2}$$

Thus, the area of this triangle is equal to

$$\text{AREA} = \frac{1}{2}xy = \left(\frac{1}{2}\right)\left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{4R\sqrt{2}}{2}\right) = \frac{8R^2}{\sqrt{3}}$$

And, finally, the planar density for this (111) plane is

$$\text{PD}_{111}(\text{BCC}) = \frac{0.5 \text{ atom}}{\frac{8R^2}{\sqrt{3}}} = \frac{\sqrt{3}}{16R^2} = \frac{0.11}{R^2}$$

7.13 *A single crystal of aluminum is oriented for a tensile test such that its slip plane normal makes an angle of 28.1° with the tensile axis. Three possible slip directions make angles of 62.4°, 72.0°, and 81.1° with the same tensile axis.*

(a) Which of these three slip directions is most favored?

(b) If plastic deformation begins at a tensile stress of 1.95 MPa (280 psi), determine the critical resolved shear stress for aluminum.

Solution

We are asked to compute the critical resolved shear stress for Al. As stipulated in the problem, $\phi = 28.1°$, while possible values for λ are 62.4°, 72.0°, and 81.1°.

(a) Slip will occur along that direction for which $(\cos \phi \cos \lambda)$ is a maximum, or, in this case, for the largest $\cos \lambda$. Cosines for the possible λ values are given below.

$$\cos(62.4°) = 0.46$$

$$\cos(72.0°) = 0.31$$

$$\cos(81.1°) = 0.15$$

Thus, the slip direction is at an angle of 62.4° with the tensile axis.

(b) From Equation 7.4, the critical resolved shear stress is just

$$\tau_{crss} = \sigma_y (\cos \phi \cos \lambda)_{max}$$

$$= (1.95 \text{ MPa})\left[\cos (28.1°) \cos (62.4°)\right] = 0.80 \text{ MPa} \quad (114 \text{ psi})$$ 7.23

7.23 *(a) From the plot of yield strength versus (grain diameter)$^{-1/2}$ for a 70 Cu–30 Zn cartridge brass, Figure 7.15, determine values for the constants σ_0 and k_y in Equation 7.7.*

(b) Now predict the yield strength of this alloy when the average grain diameter is 1.0×10^{-3} mm.

Solution

(a) Perhaps the easiest way to solve for σ_0 and k_y in Equation 7.7 is to pick two values each of σ_y and $d^{-1/2}$ from Figure 7.15, and then solve two simultaneous equations, which may be created. For example

$d^{-1/2}$ (mm) $^{-1/2}$	σ_y (MPa)
4	75
12	175

The two equations are thus

$$75 = \sigma_0 + 4 k_y$$

$$175 = \sigma_0 + 12 k_y$$

Solution of these equations yield the values of

$$k_y = 12.5 \ \text{MPa}\,(\text{mm})^{1/2} \quad \left[1810 \ \text{psi}\,(\text{mm})^{1/2}\right]$$

$$\sigma_0 = 25 \text{ MPa} \ \ (3630 \text{ psi})$$

(b) When $d = 1.0 \times 10^{-3}$ mm, $d^{-1/2} = 31.6$ mm$^{-1/2}$, and, using Equation 7.7,

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= (25 \ \text{MPa}) + \left[12.5 \ \text{MPa}\,(\text{mm})^{1/2}\right](31.6 \ \text{mm}^{-1/2}) = 420 \ \text{MPa} \ \ (61{,}000 \ \text{psi})$$

7.29 *Two previously undeformed specimens of the same metal are to be plastically deformed by reducing their cross-sectional areas. One has a circular cross section, and the other is rectangular; during deformation the circular cross section is to remain circular, and the rectangular is to remain as such. Their original and deformed dimensions are as follows:*

	Circular (diameter, mm)	***Rectangular (mm)***
Original dimensions	*15.2*	*125 × 175*
Deformed dimensions	*11.4*	*75 × 200*

Which of these specimens will be the hardest after plastic deformation, and why?

Solution

The hardest specimen will be the one that has experienced the greatest degree of cold work. Therefore, all we need do is to compute the %CW for each specimen using Equation 7.8. For the circular one

$$\%\text{CW} = \left[\frac{A_0 - A_d}{A_0}\right] \times 100$$

$$= \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \times 100$$

$$= \left[\frac{\pi\left(\frac{15.2 \text{ mm}}{2}\right)^2 - \pi\left(\frac{11.4 \text{ mm}}{2}\right)^2}{\pi\left(\frac{15.2 \text{ mm}}{2}\right)^2} \right] \times 100 = 43.8\%\text{CW}$$

For the rectangular one

$$\%\text{CW} = \left[\frac{(125 \text{ mm})(175 \text{ mm}) - (75 \text{ mm})(200 \text{ mm})}{(125 \text{ mm})(175 \text{ mm})} \right] \times 100 = 31.4\%\text{CW}$$

Therefore, the deformed circular specimen will be harder.

7.38 *The average grain diameter for a brass material was measured as a function of time at 650°C, which is tabulated below at two different times:*

Time (min)	***Grain Diameter (mm)***
30	3.9×10^{-2}
90	6.6×10^{-2}

(a) What was the original grain diameter?

(b) What grain diameter would you predict after 150 min at 650°C?

Solution

(a) Using the data given and Equation 7.9 (taking n = 2), we may set up two simultaneous equations with d_0 and K as unknowns; thus

$$(3.9 \times 10^{-2} \text{ mm})^2 - d_0^2 = (30 \text{ min})K$$

$$(6.6 \times 10^{-2} \text{ mm})^2 - d_0^2 = (90 \text{ min})K$$

Solution of these expressions yields a value for d_0, the original grain diameter, of

$$d_0 = 0.01 \text{ mm},$$

and a value for K of 4.73×10^{-5} mm^2/min

(b) At 150 min, the diameter d is computed using a rearranged form of Equation 7.9 as

$$d = \sqrt{d_0^2 + Kt}$$

$$= \sqrt{(0.01 \text{ mm})^2 + \left(4.73 \times 10^{-5} \text{ mm}^2/\text{min}\right)(150 \text{ min})} = 0.085 \text{ mm}$$