Problem 28

$\vec{V} = A/x\,\vec{i} + Ay/x^2\,\vec{j}$

Streamline:

$$\frac{dx}{u} = \frac{dy}{v} \rightarrow \frac{dx}{\cancel{A}/\cancel{x}} = \frac{dy}{\cancel{A}y/x^{\cancel{2}}} \rightarrow \int \frac{dx}{x} = \int \frac{dy}{y} \rightarrow \ln x = \ln y + C$$

$$\ln x = \ln[y\, C] \rightarrow x = y \cdot C$$

Streamline that passes through $(x,y) = (1,3)$

$$1 = 3C \rightarrow C = 1/3$$

$$\boxed{y = 3x}$$

Time required for a fluid particle to move from $x = 1m$ to $x = 3m$.

for a particle $u = \frac{dx}{dt} = \frac{A}{x}$ $\quad x\,dx = A\,dt \rightarrow$ integrating

$$\int_{x_0}^{x_1} x\,dx = A\int_{t_0}^{t_1} dt \rightarrow \int_{x_0=1}^{x_1=3} x\,dx = A\int_{t_0=0}^{t_1} dt \rightarrow At = x^2/2 \Big]_1^3$$

$$At = \frac{1}{2}(9-1) = 8/2 = 4 \quad (A = 2m^2/s) \quad 2t = 4 \rightarrow$$

$$\boxed{t = 2s}$$

Problem 2.20

$\vec{V} = axt\,\vec{i} + b\vec{j}$

pathline

$u = dx/dt = axt \rightarrow dx/ax = dt\cdot t \rightarrow \int_1^x dx/x = a\int_0^t t\,dt \longrightarrow$

$v = dy/dt = b \rightarrow dy = b\,dt \quad \int_2^y dy = \int_0^t b\,dt \longrightarrow$

for $(x_0, y_0) = (1, 2)$
at $t = 0$

$y - 2 = t \rightarrow y = \cancel{t} + 2$

$\ln x - \underset{0}{\cancel{\ln 1}} = at^2/2 - 0 \qquad \ln x = at^2/2$

$$\boxed{\begin{aligned} x &= e^{at^2/2} \\ y &= bt + 2 \end{aligned}}$$

Streakline

$u = dx/dt \rightarrow \int dx/x = a\int t\,dt \rightarrow \ln x = at^2/2 + C_1$ at an instant (unknown) $t = \tau$

$v = dy/dt \rightarrow \int dy = b\int dt \rightarrow y = bt + C_2$ $(x, y) = (1, 2)$

$$\left.\begin{aligned} x &= C_1^* e^{at^2/2} \\ y &= bt + C_2 \end{aligned}\right\} \quad \begin{aligned} 1 &= C_1^* e^{a\tau^2/2} \quad C_1^* = e^{-a\tau^2/2} \\ 2 &= b\tau + C_2 \rightarrow C_2 = 2 - b\tau \end{aligned} \quad \Big\} \Longrightarrow$$

$$\begin{aligned} x &= e^{-a\tau^2/2} \cdot e^{at^2/2} \\ y &= bt + 2 - b\tau \end{aligned} \longrightarrow \begin{aligned} x &= e^{-a\tau^2/2} \cdot e^{9a/2} \\ y &= 3b + 2 - b\tau \end{aligned} \longrightarrow x = e^{9a/2} \cdot e^{-a/2\left[3 + \frac{(2-y)}{b}\right]^2}$$

at instant $t = 3s$ the streakline is: $a = 0.3$, $b = 2$

$\tau = 3 + \frac{(2-y)}{b}$