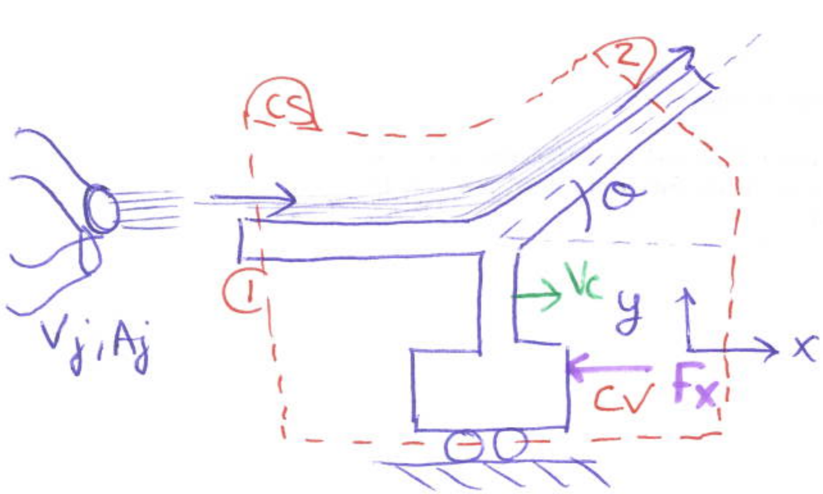
Problem 3.55



* Steady flow

$\bar{V}_{abs} = \bar{V}_{rel} + \bar{V}_{\text{moving reference frame}}$

$\bar{n}_1 = -\bar{i}$

$\bar{n}_2 = \cos\theta\,\bar{i} + \sin\theta\,\bar{j}$.

Mass Conservation: $\int_{CS} \rho(\bar{V}_{rel}\cdot\bar{n})\,dA = 0$

$\bar{V}_{rel1} = (V_j - V_c)\bar{i}$

$\bar{V}_{rel2} = V_{rel2}\cdot\bar{n}_2$

$$\int_1 \rho(\bar{V}_{rel}\cdot\bar{n})\,dA + \int_2 \rho(\bar{V}_{rel}\cdot\bar{n})\,dA = 0$$

$$-\rho(V_j - V_c)A_j + \rho V_{rel2}\cdot A_j = 0 \Rightarrow V_{rel,2} = V_j - V_c$$

Momentum Conservation: $\Sigma\vec{F} = \int_{CS} \rho\,\bar{V}_{abs}\cdot(\bar{V}_{rel}\cdot\bar{n})\,dA$

x-direction:

$$-F_x = \left[\int_1 \rho\,\bar{V}_{abs}(\bar{V}_{rel}\cdot\bar{n})\,dA + \int_2 \rho\,\bar{V}_{abs}(\bar{V}_{rel}\cdot\bar{n})\,dA\right]\cdot\vec{i}$$

$\bar{V}_{abs1} = V_j\,\bar{i}$ $\qquad \bar{V}_{rel,2} = (V_j - V_c)\bar{n}_2$

$\bar{V}_{rel1} = (V_j - V_c)\bar{i}$. $\qquad \bar{V}_{abs,2} = \bar{V}_{rel2} + \bar{V}_c = (V_j - V_c)(\cos\theta\,\bar{i} + \sin\theta\,\bar{j}) + V_c\,\bar{i}$

$\bar{V}_{abs,2} = [(V_j - V_c)\cos\theta + V_c]\bar{i} + (V_j - V_c)\sin\theta\,\bar{j}$.

F_x ↓

$$-F_x = \rho \cdot V_j \cdot [-(V_j - V_c)] A_j + \rho \left[(V_j - V_c)\cos\theta + V_c \right] \left[V_j - V_c \right] \cdot A_j$$

$$F_x = \rho A_j \cdot \left[V_j (V_j - V_c) - \left\{ (V_j - V_c)^2 \cos\theta + V_c (V_j - V_c) \right\} \right] =$$

$$F_x = \rho A_j \cdot \left[(V_j - V_c) \cdot (V_j - V_c) - (V_j - V_c)^2 \cos\theta \right] \Rightarrow$$

$$F_x = \rho A_j \cdot (V_j - V_c)^2 \cdot (1 - \cos\theta)$$