SOLUTION HW4

3.8 Three pipes steadily deliver water at 20°C to a large exit pipe in Fig. P3.8. The velocity V_2 = 5 m/s, and the exit flow rate Q_4 = 120 m^3/h. Find (a) V_1; (b) V_3; and (c) V_4 if it is known that increasing Q_3 by 20% would increase Q_4 by 10%.

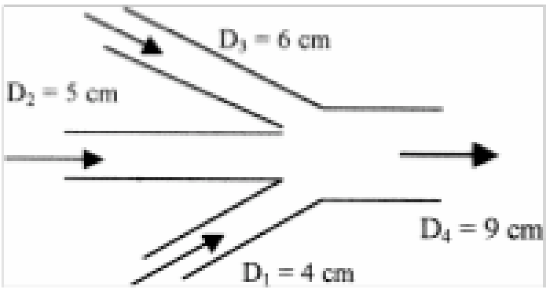


Fig. P3.8

Solution: (a) For steady flow we have $Q_1 + Q_2 + Q_3 = Q_4$, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

Since $0.2Q_3 = 0.1Q_4$, and Q_4 = (120 m^3/h)(h/3600 s) = 0.0333 m^3/s,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333 \text{ m}^3/\text{s})}{\frac{\pi}{2}(0.06^2)} = \mathbf{5.89\ m/s} \quad \textit{Ans.}\text{ (b)}$$

Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad \mathbf{V_1 = 5.45\ m/s} \quad \textit{Ans.}\text{ (a)}$$

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333 \text{ m}^3/\text{s}) = V_4(\pi)(0.06^2)/4 \qquad \mathbf{V_4 = 5.24\ m/s} \quad \textit{Ans.}\text{ (c)}$$

3.12 The pipe flow in Fig. P3.12 fills a cylindrical tank as shown. At time t = 0, the water depth in the tank is 30 cm. Estimate the time required to fill the remainder of the tank.

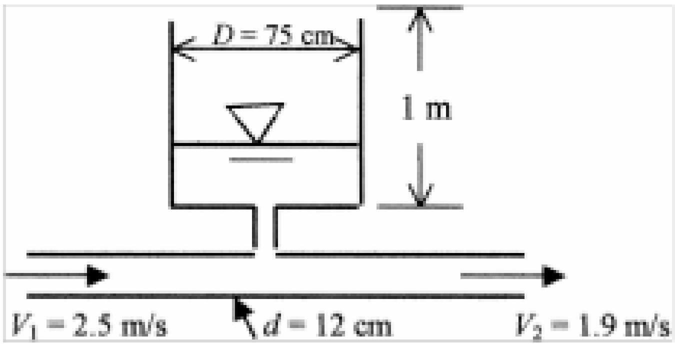


Fig. P3.12

Solution: For a control volume enclosing the tank and the portion of the pipe below the tank,

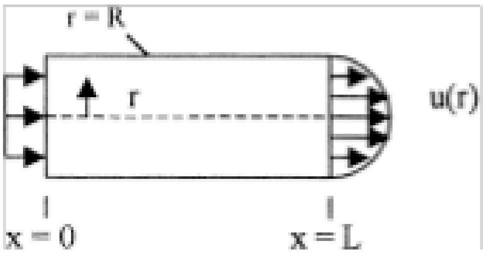
$$\frac{d}{dt}\left[\int \rho\, dv\right] + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\rho\pi R^2 \frac{dh}{dt} + (\rho AV)_{out} - (\rho AV)_{in} = 0$$

$$\frac{dh}{dt} = \frac{4}{998(\pi)(0.75^2)}\left[998\left(\frac{\pi}{4}\right)(0.12^2)(2.5-1.9)\right] = 0.0153\ m/s,$$

$$\Delta t = 0.7/0.0153 = \mathbf{46\ s} \quad \textit{Ans.}$$

3.15 Water flows steadily through the round pipe in the figure. The entrance velocity is V_o. The exit velocity approximates turbulent flow, $u = u_{max}(1 - r/R)^{1/7}$. Determine the ratio U_o/u_{max} for this incompressible flow.



Solution: Inlet and outlet flow must balance:

$$Q_1 = Q_2, \quad or: \int_0^R U_o 2\pi r\, dr = \int_0^R u_{max}\left(1-\frac{r}{R}\right)^{1/7} 2\pi r\, dr, \quad or: \quad U_o \pi R^2 = u_{max}\frac{49\pi}{60}R^2$$

Cancel and rearrange for this assumed incompressible pipe flow:

$$\frac{U_o}{u_{max}} = \mathbf{\frac{49}{60}} \quad \textit{Ans.}$$

3.17 Incompressible steady flow in the inlet between parallel plates in Fig. P3.17 is uniform, $u = U_o = 8$ cm/s, while downstream the flow develops into the parabolic laminar profile $u = az(z_o - z)$, where a is a constant. If $z_o = 4$ cm and the fluid is SAE 30 oil at 20°C, what is the value of u_{max} in cm/s?

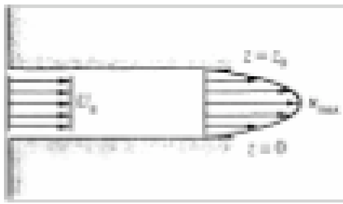


Fig. P3.17

Solution: Let b be the plate width into the paper. Let the control volume enclose the inlet and outlet. The walls are solid, so no flow through the wall. For incompressible flow,

$$0 = Q_{out} - Q_{in} = \int_0^{z_o} az(z_o - z)b\,dz - \int_0^{z_o} U_o b\,dz = abz_o^3/6 - U_o bz_o = 0, \quad \text{or:} \quad a = 6U_o/z_o^2$$

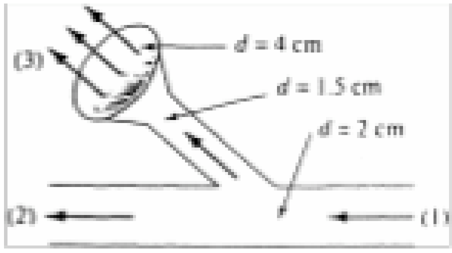
Thus continuity forces the constant a to have a particular value. Meanwhile, a is also related to the maximum velocity, which occurs at the center of the parabolic profile:

$$\text{At } z = z_o/2: \quad u = u_{max} = a\left(\frac{z_o}{2}\right)\left(z_o - \frac{z_o}{2}\right) = az_o^2/4 = (6U_o/z_o^2)(z_o^2/4)$$

$$\text{or:} \quad u_{max} = \frac{3}{2}U_o = \frac{3}{2}(8\ \text{cm/s}) = \mathbf{12\ \frac{cm}{s}} \quad \textit{Ans.}$$

Note that the result is independent of z_o or of the particular fluid, which is SAE 30 oil.

3.32 Water at 20°C flows through the piping junction in the figure, entering section 1 at 20 gal/min. The average velocity at section 2 is 2.5 m/s. A portion of the flow is diverted through the showerhead, which contains 100 holes of 1-mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead jets.



Solution: A control volume around sections (1, 2, 3) yields

$$Q_1 = Q_2 + Q_3 = 20 \text{ gal/min} = 0.001262 \text{ m}^3/\text{s}.$$

Meanwhile, with $V_2 = 2.5$ m/s known, we can calculate Q_2 and then Q_3:

$$Q_2 = V_2 A_2 = (2.5 \text{ m})\frac{\pi}{4}(0.02 \text{ m})^2 = 0.000785 \frac{\text{m}^3}{\text{s}},$$

$$\textit{hence} \quad Q_3 = Q_1 - Q_2 = 0.001262 - 0.000785 = 0.000476 \frac{\text{m}^3}{\text{s}}$$

$$\textit{Each hole carries} \quad Q_3/100 = 0.00000476 \frac{\text{m}^3}{\text{s}} = \frac{\pi}{4}(0.001)^2 V_{jet},$$

$$\textit{solve} \quad V_{jet} = \mathbf{6.06} \ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}$$

3.39 A wedge splits a sheet of 20°C water, as shown in Fig. P3.39. Both wedge and sheet are very long into the paper. If the force required to hold the wedge stationary is F = 126 N per meter of depth into the paper, what is the angle θ of the wedge?

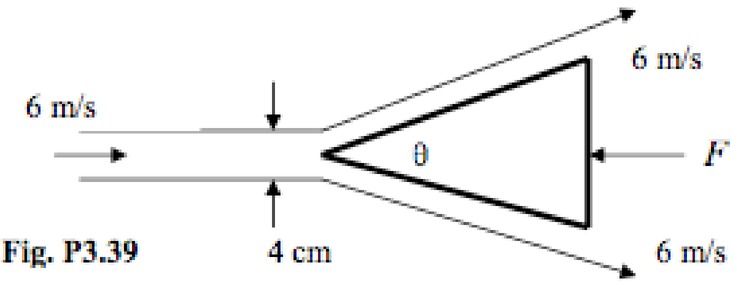


Fig. P3.39

Solution: For water take $\rho = 998\ kg/m^3$. First compute the mass flow per unit depth:

$$\dot{m}/b = \rho V h = (998\,kg/m^3)(6\,m/s)(0.04\,m) = 239.5\,kg/s-m$$

The mass flow (and velocity) are the same entering and leaving. Let the control volume surround the wedge. Then the *x*-momentum integral relation becomes

$$\Sigma F_x = -F = \dot{m}(u_{out} - u_{in}) = \dot{m}(V\cos\frac{\theta}{2} - V) = \dot{m}V(\cos\frac{\theta}{2} - 1)$$

$$or: \quad -124\,N/m = (239.5\,kg/s-m)(6\,m/s)(\cos\frac{\theta}{2} - 1)$$

$$\text{Solve} \quad \cos\frac{\theta}{2} = 0.9137\,, \quad \frac{\theta}{2} = 24^o\,, \quad \theta = \mathbf{48^o} \quad \textit{Ans.}$$

3.40 The water jet in Fig. P3.40 strikes normal to a fixed plate. Neglect gravity and friction, and compute the force *F* in newtons required to hold the plate fixed.

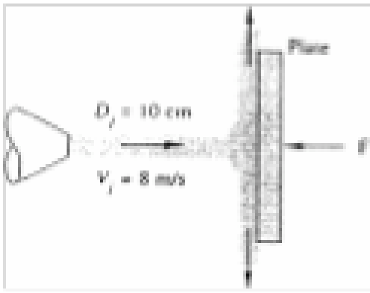


Fig. P3.40

Solution: For a CV enclosing the plate and the impinging jet, we obtain:

$$\Sigma F_x = -F = \dot{m}_{up}u_{up} + \dot{m}_{down}u_{down} - \dot{m}_j u_j$$

$$= -\dot{m}_j u_j, \quad \dot{m}_j = \rho A_j V_j$$

$$\text{Thus} \quad F = \rho A_j V_j^2 = (998)\pi(0.05)^2(8)^2 \approx \mathbf{500\ N} \leftarrow \quad \textit{Ans.}$$

3.41 In Fig. P3.41 the vane turns the water jet completely around. Find the maximum jet velocity V_o for a force F_o.

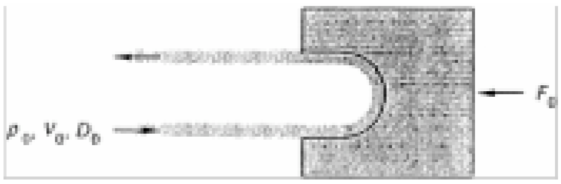


Fig. P3.41

Solution: For a CV enclosing the vane and the inlet and outlet jets,

$$\Sigma F_x = -F_o = \dot{m}_{out}u_{out} - \dot{m}_{in}u_{in} = \dot{m}_{jet}(-V_o) - \dot{m}_{jet}(+V_o)$$

$$\text{or:} \quad F_o = 2\rho_o A_o V_o^2, \quad \text{solve for} \quad V_o = \sqrt{\frac{\mathbf{F_o}}{\mathbf{2\rho_o(\pi/4)D_o^2}}} \quad \textit{Ans.}$$

3.59 A pipe flow expands from (1) to (2), causing eddies as shown. Using the given CV and assuming p = p1 on the corner annular ring, show that the downstream pressure is given by, neglecting wall friction,

$$p_2 = p_1 + \rho V_1^2\left(\frac{A_1}{A_2}\right)\left(1-\frac{A_1}{A_2}\right)$$

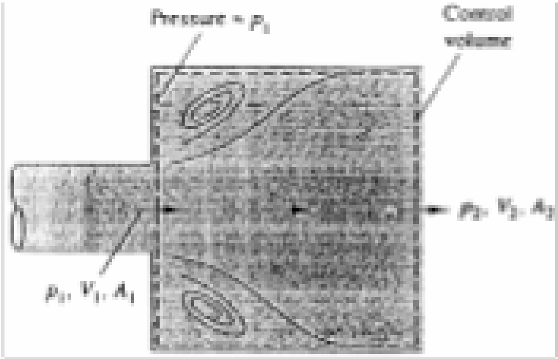


Fig. P3.59

Solution: From mass conservation, $V_1A_1 = V_2A_2$. The balance of x-forces gives

$$\sum F_x = p_1A_1 + p_{wall}(A_2 - A_1) - p_2A_2 = \dot{m}(V_2 - V_1), \quad \text{where } \dot{m} = \rho A_1V_1, \quad V_2 = V_1A_1/A_2$$

If $p_{wall} = p_1$ as given, this reduces to $\mathbf{p_2 = p_1 + \rho\frac{A_1}{A_2}V_1^2\left(1-\frac{A_1}{A_2}\right)}$ *Ans.*

3.61 A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If θ = 30°, estimate the horizontal force F needed to hold the tank stationary.

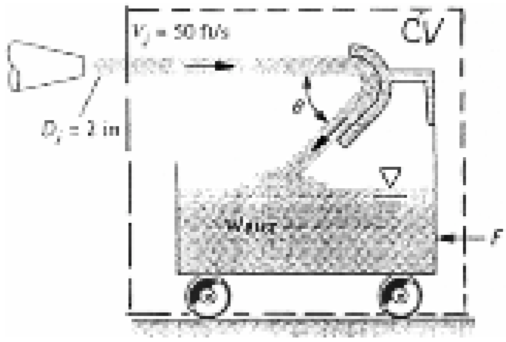


Fig. P3.61

Solution: The CV surrounds the tank and wheels and cuts through the jet, as shown. We have to *assume that the splashing into the tank does not increase the x-momentum of the water in the tank*. Then we can write the CV horizontal force relation:

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\, d\upsilon\right)_{tank} - \dot{m}_{in}u_{in} = 0 - \dot{m}V_{jet} \text{ independent of } \theta$$

Thus $F = \rho A_j V_j^2 = \left(1.94\ \frac{slug}{ft^3}\right)\frac{\pi}{4}\left(\frac{2}{12}\ ft\right)^2\left(50\ \frac{ft}{s}\right)^2 \approx \mathbf{106\ lbf}$ *Ans.*