# MAE 101B, Spring 2009
## Midterm 2
05/12/09

**Use the following fluid and solid properties:**
air: $\rho = 1.22$ kg/m$^3$; $\mu = 1.810^{-5}$ kg/ms.
polyestyrene: $\rho_P = 1050$ kg/m$^3$;

**Guidelines:**
Solve only THREE problems among problems 1, 2, 3 and 4. Indicate in your answer book which THREE problems are to be graded. Solving all four problems won't provide extra credit. Instead, you may solve the extra-credit question below for 10 points extra credit. Closed-book, closed-notes exam. Calculator is allowed. Give all formulae used in the solutions and explain your steps in each problem.

## EXTRA CREDIT. (10 points)

The streamwise velocity profile for a laminar boundary layer is given by Kármán's parabolic approximation

$$u(x, y) = U_\infty \left[ \frac{2y}{\delta(x)} - \frac{y^2}{\delta(x)^2} \right],$$

where $U_\infty$ is the velocity of the outer stream, $x$ is the streamwise coordinate, $y$ is the wall-normal coordinate and $\delta(x)$ is such that $u(x, y = \delta) = U_\infty$.

**a)** Calculate the momentum thickness $\theta$ to show that $\theta = 2\delta/15$.

**b)** Calculate the displacement thickness $\delta^*$ to show that $\delta^* = \delta/3$.

## PROBLEM 1. (100/3 points)

A solar car can be modeled as a flat plate of length $L = 3m$, width $b = 1m$ and thickness $t = 5cm$, and 4 wheels (see figure 1). The power generated by the solar panels in the car increases with their area as $\dot{W} = kbL$, with $k = 1000W/m^2$. The weight of the car is $w = \rho_c Lbt$, where $\rho_c = 3000kg/m^3$ is the density of the car. A flat-plate, zero-pressure-gradient boundary layer develops over the solar panels plate generating an unknown friction drag. The four wheels cause a total drag with $(C_D A)_{4wheels} = 0.1m^2$. The car is moving up a slope of unknown angle of inclination, $\alpha$. The speed of the car is 20 mph.

**a)** Calculate the distance from the front of the car at which the boundary layer over the solar panels becomes turbulent $x_t$. Show that this distance is small compared to the length of the car, $x_t \ll L$.

**b)** Determine the friction drag caused by the boundary layer over the solar panels assuming that it is turbulent from $x = 0$ (this assumption is justified because $x_t \ll L$).

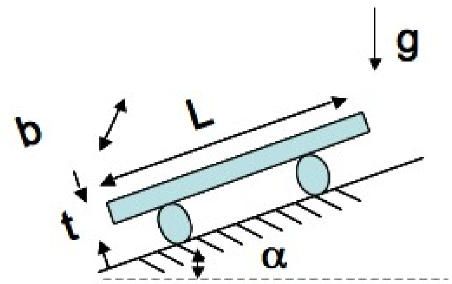


Figure 1: Solar car model

**c)** Write Newton's second law for the solar car to determine the maximum angle of inclination that it can climb at 20 mph. Hint: this angle is reached when the traction force equals zero.

**PROBLEM 2. (100/3 points)**

A rotameter is a volume flow meter that consists of a circular pipe with variable section $A(x) = A_0\sqrt{1 + x/L}$ with air flowing upwards, in which a sphere of polyestyrene or radius $r$ and density $\rho_P$ is placed as shown in figure 2. The flow velocity at the entrance of the device is $U(x = 0) = U_0$.

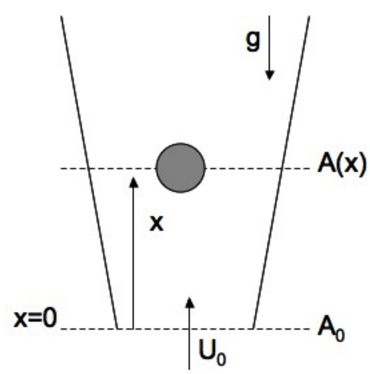


Figure 2: Rotameter

**a)** Use the mass conservation equation in the pipe to determine the flow velocity at the location $x$ where the sphere is located, $U(x)$. Neglect the area blockage caused by the sphere.

**b)** Formulate the balance of forces acting on the sphere when the air flowing upwards holds it at a constant position $x$ against its weight. Neglect buoyancy effects.

**c)** Use previous result to find an expression for $x/L$ in terms of $U_0$, $\rho_P$, $g$, $r$ and the friction coefficient $C_D$ of the sphere.

**d)** Calculate the relative position $x/L$ at which the sphere is located for given that $R = 1cm$, $U_0 = 20m/s$ and $C_D = 0.8$.

## PROBLEM 3. (100/3 points)

The streamwise velocity profile for a laminar boundary layer over a certain moving immersed object is given by Kármán's parabolic approximation

$$u(x, y) = U_\infty(x) \left[ \frac{2y}{\delta(x)} - \frac{y^2}{\delta(x)^2} \right],$$

where $U_\infty(x)$ is the velocity of the outer stream, $x$ is the streamwise coordinate, $y$ is the wall-normal coordinate and $\delta(x)$ is such that $u(x, y = \delta) = U_\infty(x)$. The geometry of the immersed object is such that

$$\delta(x) = \left( \frac{\mu^2 x}{\rho \tau_0} \right)^{1/3}, \quad U_\infty(x) = \left( \frac{\tau_0^2 x}{8 \mu \rho} \right)^{1/3},$$

where $\tau_0 > 0$ is a constant with dimensions of stress.

**a)** Show that the viscous shear stress at the wall, $\tau_w = \mu \left( \partial u / \partial y \right)_{y=0}$ is constant and equal to $\tau_0$.

**b)** Is the boundary layer under a favourable or adverse pressure gradient? Justify your answer.

## PROBLEM 4. (100/3 points)

Tour de France, Giro de Italia and Vuelta de España winner, cyclist Alberto Contador, is deciding whether to ride upright (figure 3) or racing (figure 4). Alberto can apply more power in the upright position than in the racing position, $\dot{W}_{upright} = 2KW$ and $\dot{W}_{racing} = 1.5KW$. However his drag is larger in the upright position than in the racing position, $(C_D A)_{upright} = 0.51m^2$ and $(C_D A)_{racing} = 0.3m^2$.

**a)** Write Newton's second law for Contador riding on a flat road (zero angle of inclination) to determine the traction force that he can develop in each riding position.

**b)** Calculate the critical velocity at which the traction force that Contador can develop in both positions is the same. For velocities higher than this critical velocity, it is more efficient for Contador to ride in the racing position.

**c)** Calculate the maximum velocity that Contador can reach when the rides in the upright position. Hint: this velocity is reached when the traction force equals zero.

**d)** Calculate the maximum velocity that Contador can reach when the rides in the racing position. Hint: this velocity is reached when the traction force of the car equals zero.


Figure 3: Alberto Contador in upright position


Figure 4: Alberto Contador in racing position