

# MAE 101B, Spring 2009

## Homework 3

Due Thursday, May 7, at 5:30 pm in EBU I, box outside office 2205

**Guidelines:** Please turn in a *neat* homework that gives all the formulae that you have used as well as details that are required for the grader to understand your solution. Required plots should be generated using computer software such as Matlab or Excel.

**Please refrain from copying. Refer to the course outline for what constitutes copying**

**Use the following fluid and solid properties:**

water:  $\rho = 1000 \text{ kg/m}^3$ ;  $\mu = 0.001 \text{ kg/ms}$ .

steel:  $\rho = 9000 \text{ kg/m}^3$ ;

1. Water flows at velocity  $U$  over a thin flat plate. A boundary layer with thickness  $\delta$  forms on the plate.
  - a) Assume two-dimensional steady incompressible flow with zero pressure gradient. Give the governing equations for the boundary layer as well as the boundary conditions.
  - b) Rewrite the result obtained in part (a) in term of stream function  $\psi$ . State the boundary conditions for  $\psi$ .
  - c) Show that for  $\psi = U\delta f(\eta)$  the PDEs in part (b) can be reduced to a single ODE with constant coefficients. Here,  $f' = u/U$  is an explicit function of  $\eta = y/\delta(x)$ . Give the boundary conditions for the ODE.
2.
  - a) Show that the curvature of the velocity profile  $d^2u/dy^2$  must be zero at the wall in a boundary layer with zero pressure gradient.
  - b) The parabolic profile assumption in a laminar boundary layer can be replaced by the following polynomial profile (which satisfies the zero curvature requirement) in the Karman momentum integral analysis:

$$\frac{u}{U} = 2\frac{y}{\delta} - 2\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}$$

Obtain the shape factor,  $H$ . Obtain an expression that relates  $\delta/x$  to  $Re_x$ .

3. A missile is launched from the ground with a net upward acceleration of  $2m/s^2$ . Assume that the boundary layer on the fin surface can be approximated as that on a flat plate.
  - a) What is the expected length of the laminar boundary layer  $L_{lam}$  on the fin when the missile hits 5 km elevation?
  - b) What is  $L_{lam}$  at 10 km elevation?

*Note: see tables A2 and A6 from the book for air properties as a function of the altitude*

4. Calculate the power needed to launch a cone-shaped torpedo, of length  $L = 3$  m and opening angle  $\theta = 20^\circ$ , at an initial speed of 25 m/s from a submarine immersed in water.
5. Assuming high Reynolds number regime, calculate the terminal velocity  $U_\infty$  of a steel sphere of radius  $R = 1$  cm with a measured drag coefficient  $C_D = 0.8$  falling down in water. After having done your calculations, check that the high Reynolds number assumption is correct.

**Ungraded problems** From text. 7.8, 7.27, 7.59, 7.66.