MAE 101B, Spring 2009
Equation Sheet for Final

Perfect Gas Model

$$P = \rho R T$$

$R = 287 J/kg\ K$

Entropy and Enthalpy:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = c_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}, \qquad h_2 - h_1 = c_p (T_2 - T_1)$$

Definition of Mach Number

$$Ma = \frac{V}{c},$$

with c speed of sound, for perfect gas model $c = \sqrt{\gamma R T}$

Isentropic Flow

$$\Delta s = 0, \quad \frac{T_1}{T_2} = \left(\frac{P_1}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Stagnation Properties on Isentropic flow

$$\frac{T_0}{T} = \left(\frac{c_0}{c}\right)^2 = 1 + \frac{\gamma - 1}{2} Ma^2, \qquad \frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{\frac{1}{\gamma-1}}$$

Critical values in the sonic point:

$$\frac{P^*}{P} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{\rho^*}{\rho} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}} \qquad \frac{T^*}{T} = \frac{2}{\gamma + 1} \qquad \frac{c^*}{c} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}}$$

Choked Nozzle

$$Ma_t^* = 1, \frac{m^*}{m} = \frac{A}{A^*} = \frac{1}{Ma} \left(\frac{2}{\gamma + 1}[1 + \frac{\gamma - 1}{2} Ma^2]^{\frac{\gamma+1}{2(\gamma-1)}}\right)$$

Nozzle in critical conditions:

$$Ma_t^* = 1\ (throat) \quad P_e = P_{atm}\, and\, M_e < 1\, [at\, exit,\ subsonic]$$

Nozzle in design conditions:

$$Ma_t^* = 1\ (throat) \quad P_e = P_{atm}\, and\, M_e > 1\, [at\, exit,\ supersonic]$$

Normal Shock Waves:

Tables for properties before and after normal shock waves will be provided in the exam,

$$\frac{P_2}{P_1} = \frac{1}{\gamma+1}\left[2\gamma Ma_1^2 - (\gamma-1)\right], \qquad \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)Ma_1^2}{(\gamma-1)Ma_1^2+2}$$

$$\frac{T_2}{T_1} = \left[2+(\gamma-1)Ma_1^2\right]\frac{2\gamma Ma_1^2-(\gamma-1)}{(\gamma+1)^2 Ma_1^2} \qquad Ma_2 = \frac{(\gamma-1)Ma_1^2+2}{2\gamma Ma_1^2-(\gamma-1)}$$

Oblique Shock Waves:

θ= deflection angle

β= shock angle

$$Ma_{n1} = \frac{V_{n1}}{a_1} = Ma_1 \sin\beta \qquad Ma_{n2} = \frac{V_{n2}}{a_2} = Ma_2 \sin(\beta-\theta)$$

$$\tan\theta = \frac{2\cot\beta(Ma_1^2\sin^2\beta - 1)}{Ma_1^2(\gamma+\cos 2\beta)+2}$$