MAE 101B, Spring 2009

Equation Sheet for Midterm 2

05/12/09

Viscous Shear Stress on a Plane Wall (defined by $y = 0$)

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}.$$

Flat-Plate Boundary Layer Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} = \rho U \frac{\partial U}{\partial x} + \frac{\partial^2 u}{\partial y^2}.$$

Bernouilli Equation

$$p(x) + \frac{\rho U(x)^2}{2} = \text{constant} = p_\infty + \frac{\rho U_\infty^2}{2}.$$

Friction and Drag Coefficients for a Zero-Pressure-Gradient, Flat-Plate, Laminar Boundary Layer

$$C_f(x) = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.664}{\sqrt{Re_x}},$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 b L} = \frac{1.328}{\sqrt{Re_x}}.$$

Transition from Laminar to Turbulent Boundary Layer

$$Re_{x_t} \approx 10^6$$

Friction and Drag Coefficients for a Zero-Pressure-Gradient, Flat-Plate, Turbulent Boundary Layer

$$C_f(x) = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.027}{Re_x^{1/7}},$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 b L} = \frac{0.031}{Re_L^{1/7}}.$$

Integral Measures of Boundary Layer Thickness

Momentum Thickness

$$\theta(x) = \int_0^{\delta(x)} \frac{u(x,y)}{U(x)} \left[1 - \frac{u(x,y)}{U(x)}\right] dy.$$

Displacement Thickness

$$\delta^*(x) = \int_0^{\delta(x)} \left[1 - \frac{u(x,y)}{U(x)}\right] dy.$$

Mass Conservation Equation in a Pipe

$$U(x)A(x) = \text{constant} = U(x=0)A(x=0)$$

Newton's Second Law Applied to a Vehicle

$$ma = \frac{\dot{W}}{U} - D - w\sin\alpha,$$

where:

ma mass times acceleration of the vehicle, $\dot{W}$ is the vehicle power, D is the drag generated by the fluid, w is the weight of the vehicle, α is the slope of the road ($\alpha > 0$ if the road goes upwards).

Newton's Second Law Applied to a particle suspended in a vertical stream

$$ma = D - w,$$

where:

ma is mass times acceleration of the particle, D is the drag generated by the fluid, w is the weight of the particle.