MAE 118C Spring Quarter 2008
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PRACTICE FINAL EXAM

1. The coulomb potential is given as $\phi(r) = \frac{Zq_p}{4\pi\varepsilon_0 r}$ for an ion of charge Zq_p. We wish to fuse this nucleus with another nucleus of charge Z'q. The effective radius of the nuclear force is r_n. Estimate the minimum kinetic energy required for fusion to occur.

2. A deuterium ion beam with current I, cross-sectional area A, and energy E_0 (eV) is fired at a slab-shaped target of thickness t containing cold Tritium at density N_T. Suppose that somehow the fusion reaction cross section was modified such that σ_f=constant independent of the incident particle energy (note that this isn't possible but suspend such considerations for the purposes of this problem).
    a. Find the probability that a beam deuterium ion will undergo fusion while moving through the target.
    b. Given these assumptions, find the amount of fusion power produced by this arrangement.
    c. Estimate the power gain, G, defined as the power output divided by the power input. You may neglect elastic scattering in this analysis.

3. Suppose a particular nucleus is composed of N neutrons and Z protons. The rest mass of the nucleus, M, is less than the sum of the neutron and proton masses, i.e. $M < Nm_N + Zm_p$, where m_N, m_p denote the rest mass of a neutron or proton. On average, how much work must be done to remove one nuclear particle from this nucleus?

4. A beam of particles of initial intensity I_0 collides with a semi-infinite slab target of particles. The particle slab begins at x=0, is located at x>0, has a thickness t, and the beam travels in the +x-direction. The slab is composed of two species of particles with densities n_1 and n_2 per unit volume respectively, and both species are distributed uniformly throughout the slab. The beam particles interact with both types of target particles with interaction probabilities represented by cross-sections σ_1 and σ_2 respectively. Find the un-interacted beam intensity at x=t.

5. Suppose you have N point sources of neutrons, each with strength S, located at positions $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, …, $\mathbf{x}_N$, each emitting mono-energetic neutrons into vacuum. Find the flux of neutrons at a point $\mathbf{x}$ that is not co-incident with any of the point sources.

6.

a. Use a control volume analysis to find the neutron continuity equation, making sure to account for neutron production, absorption, gradients, and time variations.
b. Assume that the vector neutron current density obeys a Fick's Law diffusion model derive the one-group time-dependent neutron diffusion equation.

7. The condition for reactor criticality depends on the factor

$$k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a} > 1$$

where B satisfies the equation

$$\nabla^2\phi + B^2\phi = 0 .$$

Find an expression for the minimum macroscopic fission cross section necessary for critical operation of an infinite, bare slab reactor of thickness a.

8. A slab-shaped neutron moderator with uniform macroscopic absorption cross-section Σ_a contains a planar source of neutrons at x=0. The slab is centered at x=0, has a finite thickness 2a, and the source has a strength S neutrons/(unit area – unit time). The diffusion equation is given as $D\nabla^2\phi - \Sigma_a\phi = -S$. Find the probability that a neutron emitted by the source will escape the slab. What happens to this probability as the thickness becomes very large?

9. In magnetic fusion the plasma pressure is balanced by a magnetic force which is proportional to the gradient of the magnetic energy density, $B^2 / 2\mu_0$.

(a) If you have a cylindrical plasma of radius a, a pressure profile p=p(r), and an external magnetic field B, find the reduction of the magnetic field at r=0.
(b) Suppose you wish to confine a fusion plasma with temperature T and density N particles/m^3. If the maximum beta is β_{max}, what strength of magnetic field is needed ?