MAE 118C Spring Quarter 2008
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MIDTERM – CLOSED BOOK CLOSED NOTES NO CALCULATORS OR OTHER ELECTRONIC DEVICES PERMITTED.
ALL PROBLEMS WEIGHTED EQUALLY.

1. Two nuclear particles with rest mass M_a and M_b and incident kinetic energies E_a and E_b interact to yield two other nuclear particles with rest mass M_c and M_d. The reaction is endothermic, i.e. the masses are such that $M_c + M_d > M_a + M_b$. What minimum amount of incident kinetic energy, $E_a + E_b$ must the incident particles have to allow this reaction to occur?

2. Suppose a particular nucleus is composed of N neutrons and Z protons. The rest mass of the nucleus, M, is less than the sum of the neutron and proton masses, i.e. $M < Nm_N + Zm_p$, where m_N, m_p denote the rest mass of a neutron or proton. How much work must be done to remove one nuclear particle from this nucleus?

3. A monoenergetic beam of particles with intensity I_0 is incident on a uniform slab target of thickness d. The number density of nuclei in the target is N and the microscopic cross section for interactions between beam particles and target nuclei is given as σ.
   a. Write down the macroscopic cross section in terms of N and σ.
   b. Derive the differential equation for the intensity of the un-interacted beam intensity at a depth x into the target.
   c. Solve this equation, and use that solution to find the fraction of the beam that interacts with the nuclei in the slab.

4. Two point sources of neutrons, each with strength S, located at y=+b and y=-b emit monoenergetic neutrons into vacuum. Find the flux of neutrons at any point.

5. A slab-shaped neutron moderator with uniform macroscopic absorption cross-section Σ_a contains a planar source of neutrons at x=0. The slab is centered at x=0, has a finite thickness 2a, and the source has a strength S neutrons/(unit area – unit time). The diffusion equation is given as $D\nabla^2\phi - \Sigma_a\phi = -S$. Find the probability that a neutron emitted by the source will be absorbed within the slab.

1. $$[(M_c + M_d) - (M_a + M_b)]c^2 = E_a + E_b$$

2. $$\frac{[(N m_N + Z m_p) - M]c^2}{N+Z}$$ = work to remove one nuclear particle (average)

3.

a) $\Sigma = N\sigma$

b) $dI = -N\sigma I x \, dx = -I\Sigma dx$

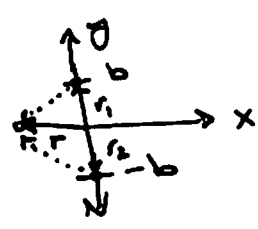
c) $$\int_0^x \frac{dI}{I} = \int_0^x -\Sigma \, dx$$

$$I(x) = I_0 e^{-\Sigma x}$$

$$\frac{I(d)}{I(0)} = \frac{I_0 e^{-\Sigma d}}{I_0} = e^{-\Sigma d}$$ fraction that doesn't interact

$$1 - e^{-\Sigma d}$$ = fraction that interacts.

4. $$\phi = \frac{S}{4\pi r^2}$$



$r_1 = b\hat{y}$

$r_2 = -b\hat{y}$

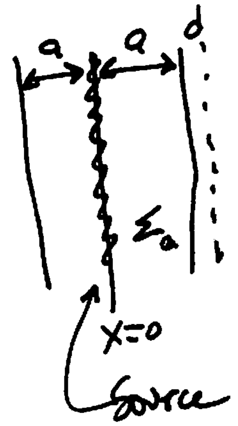
$r - r_1$

$r - r_2$

$$\phi_T = \left(\frac{S}{4\pi(\vec{r} - \vec{r}_1)^2}\right) + \left(\frac{S}{4\pi(\vec{r} - \vec{r}_2)^2}\right)$$

$$\phi_T = \frac{S}{4\pi(\vec{r} - b\hat{y})^2} + \frac{S}{4\pi(\vec{r} + b\hat{y})^2}$$

5.



$$D\nabla^2\phi - \Sigma_a\phi = -S$$

$$\phi = Ae^{-x/L} + Ce^{x/L}$$

$$\phi(a+d) = 0$$

$$0 = Ae^{-(a+d)/L} + Ce^{(a+d)/L}$$

$$C = -Ae^{-2(a+d)/L}$$

$$\phi = A\left[e^{-x/L} - e^{x/L - 2(a+d)/L}\right]$$

$$A = \frac{SL}{2D}\left(1 + e^{-2(a+d)/L}\right)^{-1}$$

absorption rate $= \int_V \Sigma_a \phi \, dV$

production rate $= \int_V S \, dV$

absorption percentage $= \dfrac{\int_V \Sigma_a \phi \, dV}{\int_V S \, dV} = \int_V \dfrac{\Sigma_a \phi}{S} dV$