Homework 1
MAE 118C

Problems 1, 4, 5, 6, 12, 14, 15, 16, 18, 27, 34, 35, 37, 38, 39, 48, 51, 52, 53, 55 of Chapter 2, Lamarsh & Baratta

1.

a) ^{7}Li Nuetrons = 4
Protons = 3

b) ^{24}Mg Nuetrons = 12
Protons = 12

c) ^{135}Xe Nuetrons = 81
Protons = 54

d) ^{209}Bi Nuetrons = 126
Protons = 83

e) ^{222}Rn Nuetrons = 136
Protons = 86

4.

a) H_2

$$.01\cdot(99.985\cdot 1.007825 + 0.015\cdot 2.01410)\cdot 2 = 2.016$$

b) H_2O

$$.01\cdot(99.985\cdot 1.007825 + 0.015\cdot 2.01410)\cdot 2 + .01\cdot(99.759\cdot 15.99492 + .037\cdot 16.99913 + .204\cdot 17.99916) = 18.($$

= 18.015

c) H_2O_2

$$.01\cdot(99.985\cdot 1.007825 + 0.015\cdot 2.01410)\cdot 2 + .01\cdot(99.759\cdot 15.99492 + .037\cdot 16.99913 + .204\cdot 17.99916)\cdot 2 = 3$$

= 34.015

5.

Molecular weight of approximately 2: $.99985\cdot .99985\cdot 100 = 99.970002$ percent

Molecular weight of approximately 3: $.99985\cdot .00015\cdot 100 = 0.015$ percent

Molecular weight of approximately 4: $.00015\cdot .00015\cdot 100 = 2.25\times 10^{-6}$ percent

6.

$$.01\cdot(.0057\cdot 234.0409 + .72\cdot 235.0439 + 99.27\cdot 238.0508) = 238.019$$

12.

$$R_1 := 1.25\times 10^{-12}\mathrm{m}\cdot 238^{\frac{1}{3}} = 7.746\times 10^{-12}\mathrm{m} \quad R_2 := 2\cdot 10^{-10}\cdot\mathrm{m} \quad \left(\frac{R_1}{R_2}\right)^3 = 5.81055\times 10^{-5}$$

14. Using the numbers from problem 12 to find the density of the nuclei,

$$\frac{4}{3}\cdot\pi\cdot R_1^{\,3} = 4.64\cdot 10^{-34}\cdot m^3$$

$$238.0508\cdot 1.66057\cdot 10^{-24}\cdot gm = 3.953\cdot 10^{-25}\cdot kg$$

Density = $\dfrac{3.953\cdot 10^{-25}\cdot kg}{4.64\cdot 10^{-34}\cdot m^3} = 8.519\times 10^{8}\cdot\dfrac{kg}{m^3}$

$$\frac{6\cdot 10^{24} kg}{8.519\cdot 10^{8}\,\frac{kg}{m^3}} = 7.043\times 10^{15}\cdot m^3 \qquad \left[\frac{3}{4\cdot\pi}\cdot\left(7.043\cdot 10^{15}\cdot m^3\right)\right]^{\frac{1}{3}} = 45.761\cdot km$$

The earth would have a redius of ~45 kms

15.

$$E := 1gm\cdot c^2$$

$$c = 2.998\times 10^{8}\,\frac{m}{s}$$

$$E = 8.988\times 10^{13} J$$

$\dfrac{E}{3\cdot 10^{7}\,\frac{J}{kg}} = 2.996\times 10^{6} kg$ of coal is equivalent to the conversion of 1g of mass converted to energy.

16.

$$N_A := 0.6022045\cdot 10^{24}$$

$\dfrac{1gm}{235.0439gm}\cdot N_A = 2.562\times 10^{21}$ number of ^{235}U atoms in 1g

$1.60219\cdot 10^{-19}\cdot J = 1eV \qquad MeV := 1.60219\cdot 10^{-19}\cdot 10^{6}\cdot J$

$$2.562\cdot 10^{21}\cdot 200\cdot MeV = 8.21\times 10^{10} J$$

$$2.562\cdot 10^{21}\cdot 200\cdot MeV = 2.28\times 10^{4}\cdot kW\cdot hr$$

$$hr = 3.6\times 10^{3} s$$

$$day = 8.64\times 10^{4} s$$

$$2.562\cdot 10^{21}\cdot 200\cdot MeV = 0.95\cdot MW\cdot day$$

18.

a) Final kinetic energy = 5 MeV

b) Total energy = KE + Rest energy = $E_{total} := 5\text{MeV} + 0.511\text{MeV} = 5.511\cdot\text{MeV}$

c) Rest mass = $\dfrac{E_{total}}{c^2} = 9.824331807923402 \times 10^{-30}\,\text{kg}$

The percision is a little excessive, but it's the only way I could get matlab to display something other than zero for small numbers.

27.

	Frist Excited	Second Excited	Thrid Excited
Ground State	0.158MeV	0.208MeV	0.403MeV
Frist Excited	n/a	0.050MeV	0.245MeV
Second Excited	n/a	n/a	0.195MeV

34.

$PV = NkT$ 1 gram of ^{226}Ra Assuming a fixed temperature

$V_1 := 1.2\text{cm}^3$ $T_1 := 300\text{K}$

$3.7\cdot10^{10}$ disintegratios per second equals 1 curie which is the rate for ^{226}Ra by definition.

$N_A = 6.022 \times 10^{23}$

$k := 1.3807\cdot10^{-23}\,\dfrac{J}{K}$ $N_1 := 3.7\cdot10^{10}\cdot\dfrac{1}{s}$

a) $dP := \dfrac{N_1\cdot k\cdot T_1}{V_1} = 1.277 \times 10^{-4}\cdot\dfrac{Pa}{s}$ The rate of pressure change in the capsule.

b) $P := \dfrac{N_1\cdot 10\text{yr}\cdot k\cdot T_1}{V_1} = 0.398\cdot\text{atm}$ The pressure due to He particles.

35.

$\dfrac{1\text{MW}}{5.305\text{MeV}} = 1.177 \times 10^{18}\,\dfrac{1}{s}$ decays per second for 1 MW

$138\text{day} = 1.192 \times 10^{7}\,s$

$t := 1.44\cdot138\text{day} = 1.717 \times 10^{7}\,s$

$\dfrac{1}{t} = 5.824 \times 10^{-8}\,\dfrac{1}{s}$

$$\frac{1.177\cdot 10^{18}\,\frac{1}{s}}{5.824\cdot 10^{-8}\,\frac{1}{s}} = 2.021\times 10^{25} \qquad \frac{2.021\cdot 10^{25}}{N_A}\cdot 210\text{gm} = 7.048\ \text{kg}$$

Another method for 35, the solutions are very similar.

$t := 1\text{s}$

$T_{.5} := 138\cdot\text{day}$

$$\text{Lambda} := \frac{0.693}{T_{.5}} = 5.812\times 10^{-8}\,\frac{1}{s}$$

$$n_r := \frac{1\text{MW}}{5.305\text{MeV}}\cdot t = 1.177\times 10^{18}$$

$$n_0 := \frac{n_r}{1 - e^{-\text{Lambda}\cdot t}} = 2.024\times 10^{25} \qquad \frac{2.024\times 10^{25}}{N_A}\cdot 210\text{gm} = 7.058\ \text{kg}$$

37.

U^{235}	$T_{.5} := 7.13\cdot 10^{8}\text{yr}$	a/0 =	0.72	$\lambda_1 := \frac{0.693}{T_{.5}} = 3.0799879237148966\times 10^{-17}\,\frac{1}{s}$
U^{238}	$T_{.5} := 4.51\cdot 10^{8}\text{yr}$	a/0 =	99.27	$\lambda_2 := \frac{0.693}{T_{.5}} = 4.869249200906255\times 10^{-17}\,\frac{1}{s}$

When did a/0=3.0 for U^{235}?

$$\frac{n_1}{n_2} = \frac{n_{01}}{n_{02}}\cdot\frac{e^{-\lambda_1\cdot t}}{e^{-\lambda_2\cdot t}} \qquad \frac{n_1}{n_2} = \frac{0.72\cdot 100}{99.27} = .725 \qquad \frac{n_{01}}{n_{02}} = 3.0$$

solve for t in the above equation,

$$t = \frac{\ln\left(\frac{.725}{3}\right)}{\lambda_1 - \lambda_2} = 2.515\times 10^{9}\cdot\text{yr}$$ long time

38.

$$\alpha = \alpha_{0e}^{-\lambda\cdot t} + R\left(1 - e^{-\lambda\cdot t}\right) \qquad t = T_{.5}$$

$\alpha_0 := 0$

$\lambda\cdot t = 0.693$

$$xR = R\left(1 - e^{-\lambda\cdot t}\right) \qquad x = \left(1 - e^{-0.693}\right) = .5$$ 50% of the saturation activity of Y

39.

A ---> B ---> C --->

No atoms of B at t=0

a)

$$\alpha_B = \frac{\alpha_{A0}\cdot\lambda_B}{\lambda_B - \lambda_A}\cdot\left(e^{-\lambda_A\cdot t} - e^{-\lambda_B\cdot t}\right)$$

$$\frac{d}{dt}\alpha_B = \frac{\alpha_{A0}\cdot\lambda_B}{\lambda_B - \lambda_A}\cdot\left(-\lambda_A\cdot e^{-\lambda_A\cdot t} + \lambda_B\cdot e^{-\lambda_B\cdot t}\right) = 0$$

$$\lambda_A\cdot e^{-\lambda_A\cdot t} = \lambda_B\, e^{-\lambda_B\cdot t}$$

$$\frac{\lambda_A}{\lambda_B} = e^{t\cdot(\lambda_A-\lambda_B)} \qquad \ln\left(\frac{\lambda_A}{\lambda_B}\right) = t\cdot(\lambda_A - \lambda_B) \qquad \ln\left(\frac{\lambda_B}{\lambda_A}\right) = t\cdot(\lambda_B - \lambda_A)$$

$$t_m = \frac{\ln\left(\frac{\lambda_B}{\lambda_A}\right)}{\lambda_B - \lambda_A}$$

b)

$$\alpha_B = \frac{\alpha_{A0}\cdot\lambda_B}{\lambda_B - \lambda_A}\cdot\left(e^{-\lambda_A\cdot t} - e^{-\lambda_B\cdot t}\right) \qquad \alpha_A = \alpha_{A0}\cdot e^{-\lambda_A\cdot t}$$

$$\frac{\alpha_B}{\alpha_A} = \frac{\lambda_B}{\lambda_B - \lambda_A}\cdot\left[1 - e^{t\cdot(\lambda_A-\lambda_B)}\right]$$

This shows that the activities are equal at t_m

$$\frac{\lambda_B}{\lambda_B - \lambda_A}\cdot\left[1 - e^{\left(\frac{\ln\left(\frac{\lambda_B}{\lambda_A}\right)}{\lambda_B-\lambda_A}\right)\cdot(\lambda_A-\lambda_B)}\right] \rightarrow \frac{\lambda_B\cdot\left(\frac{\lambda_A}{\lambda_B} - 1\right)}{\lambda_A - \lambda_B} \qquad \frac{\lambda_B\cdot\left(\frac{\lambda_A - \lambda_B}{\lambda_B}\right)}{\lambda_A - \lambda_B} = 1$$

$$\frac{d}{dt}\alpha_A = -\lambda_A\cdot\alpha_{A0}\cdot e^{-\lambda_A\cdot t_m} = -\lambda_B\cdot\alpha_{A0}\cdot e^{\frac{-\lambda_A}{\lambda_B-\lambda_A}}$$

which is negative for all possible λ_As and λ_Bs

The slope of α_A is negative and α_B is zero at t_m,
this means $\alpha_A > \alpha_B$ for $t < t_m$ and $\alpha_A < \alpha_B$ for $t > t_m$

48.

$\Delta(3H) = 14.95\text{MeV}$

$\Delta(2H) = 13.14\text{MeV}$

$\Delta(n) = 8.07\text{MeV}$

$\Delta(4He) = 2.42\text{MeV}$

$Q = (\Delta_a + \Delta_b) - (\Delta_c - \Delta_d)$

$Q := (14.95\text{MeV} + 13.14\cdot\text{MeV}) - (8.07\cdot\text{MeV} + 2.42\cdot\text{MeV}) = 17.6\cdot\text{MeV}$

51.

$^{6}Li(\alpha,p)^{9}Be$ $\qquad m_p := 1.67265\cdot10^{-27}\cdot\text{kg} \qquad m_n := 1.67495\cdot10^{-27}\cdot\text{kg}$

a)

$M_{Be} := \dfrac{9.0122\cdot\text{gm}}{N_A} = 1.4965348150005522\times10^{-26}\,\text{kg}$

$m_p\cdot c^2 = 938.28\cdot\text{MeV}$

$M_{Li} := \dfrac{6.015\text{gm}}{N_A} = 9.98830131624722\times10^{-27}\,\text{kg}$

$M_{He} := \dfrac{4.0026\text{gm}}{N_A} = 6.6465793596693485\times10^{-27}\,\text{kg}$

$(4\cdot m_p + 5\cdot m_n - M_{Be})\cdot c^2 = 56.096\cdot\text{MeV} \qquad \dfrac{56.09}{9} = 6.232$

$(3\cdot m_p + 3\cdot m_n - M_{Li})\cdot c^2 = 30.571\cdot\text{MeV} \qquad \dfrac{30.571}{7} = 4.367$

$(2\cdot m_p + 2\cdot m_n - M_{He})\cdot c^2 = 27.274\cdot\text{MeV} \qquad \dfrac{27.274}{4} = 6.819$

b)

$Q := (30.571\cdot\text{MeV} + 27.274\cdot\text{MeV}) - 56.096\cdot\text{MeV} = 1.749\cdot\text{MeV}$

52.

$$M_{2H} := \frac{2.014gm}{N_A} \qquad \left(m_p + m_n - M_{2H}\right)\cdot\frac{c^2}{2} = 0.903\cdot MeV$$

$$M_{4He} := \frac{4.0026\cdot gm}{N_A} \qquad \left(2\cdot m_p + 2\cdot m_n - M_{4He}\right)\cdot\frac{c^2}{4} = 6.818\cdot MeV$$

$$M_{12C} := \frac{12.0107\cdot gm}{N_A} \qquad \left(6\cdot m_p + 6\cdot m_n - M_{12C}\right)\cdot\frac{c^2}{12} = 6.593\cdot MeV$$

$$M_{51V} := \frac{50.9415\cdot gm}{N_A} \qquad \left(23\cdot m_p + 28\cdot m_n - M_{51V}\right)\cdot\frac{c^2}{51} = 8.556\cdot MeV$$

$$M_{138Ba} := \frac{137.9052\cdot gm}{N_A} \qquad \left(56\cdot m_p + 82\cdot m_n - M_{138Ba}\right)\cdot\frac{c^2}{138} = 8.185\cdot MeV$$

$$M_{235U} := \frac{235.0439\cdot gm}{N_A} \qquad \left(92\cdot m_p + 143\cdot m_n - M_{235U}\right)\cdot\frac{c^2}{235} = 7.39\cdot MeV$$

53.

$$m_H := 1.007825 \qquad m_n := 1.008664904$$

$$m_{2H} := 2.014 \qquad \left(1\cdot m_H + 1\cdot m_n - m_{2H}\right)\cdot\frac{931.5}{2}MeV = 1.16\cdot MeV$$

$$m_{4He} := 4.0026 \qquad \left(2\cdot m_H + 2\cdot m_n - m_{4He}\right)\cdot\frac{931.5}{4}MeV = 7.075\cdot MeV$$

$$m_{12C} := 12.0107 \qquad \left(6\cdot m_H + 6\cdot m_n - m_{12C}\right)\cdot\frac{931.5}{12}MeV = 6.85\cdot MeV$$

$$m_{51V} := 50.9415 \qquad \left(23\cdot m_H + 28\cdot m_n - m_{51V}\right)\cdot\frac{931.5}{51}MeV = 8.787\cdot MeV$$

$$m_{138Ba} := 137.9052 \qquad \left(56\cdot m_H + 82\cdot m_n - m_{138Ba}\right)\cdot\frac{931.5}{138}MeV = 8.394\cdot MeV$$

$$m_{235U} := 235.0439 \qquad \left(92\cdot m_H + 143\cdot m_n - m_{235U}\right)\cdot\frac{931.5}{235}MeV = 7.591\cdot MeV$$

The mass formula gives slightly larger results for the binding energy per nucleon.

55.

Eq. 2.52 $$N(E) = \frac{2\cdot\pi N}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}$$

$$\ln(N(E)) = \ln\left[\frac{2\cdot\pi\,N}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}\right] = \ln(2\cdot\pi\cdot N) - \frac{3}{2}\cdot\ln(\pi\cdot k\cdot T) + \frac{1}{2}\cdot\ln(E) - \frac{E}{kT}$$

$$\frac{d}{dE}(\ln(N(E))) = \frac{\frac{d}{dE}N(E)}{N(E)} = 0 - 0 + \frac{1}{2}\cdot\frac{1}{E} - \frac{1}{kT} = 0$$

$$\frac{1}{2}\cdot\frac{1}{E} = \frac{1}{kT} \qquad E = \frac{1}{2}\cdot kT$$