MAE 118C Spring Quarter 2008
Professor G.R. Tynan

FINAL EXAM

CLOSED BOOK CLOSED NOTES. NO CALCULATORS OR ELECTRONIC DEVICES OF ANY KIND ARE PERMITTED.

1. The coulomb potential is given as $\phi(r) = \dfrac{Zq_p}{4\pi\varepsilon_0 r}$ for an ion of charge Zq_p. We wish to fuse this nucleus with another nucleus of charge $Z'q_p$. The force on the second nucleus is $F_r = -Z'q\dfrac{\partial \phi}{\partial r}$. If the effective radius of the nuclear force is r_n, find the work that would have to be done on the particles to bring them from a large separation distance to a distance close enough for fusion to occur.

2. Suppose a particular nucleus is composed of N neutrons and Z protons. The rest mass of the nucleus, M, is less than the sum of the neutron and proton masses, i.e. $M < Nm_N + Zm_p$, where m_N, m_p denote the rest mass of a neutron or proton. On average, how much work must be done to remove one nuclear particle from this nucleus?

3. A beam of particles of initial intensity I_0 collides with a slab target of particles. The particle slab begins at x=0, is located at x>0, has a thickness t, and the beam travels in the +x-direction. The slab is composed of two species of particles with densities n_1 and n_2 per unit volume respectively, and both species are distributed uniformly throughout the slab. The beam particles interact with both types of target particles with interaction probabilities represented by cross-sections σ_1 and σ_2 respectively.

    a. Find the un-interacted beam intensity at x=t.
    b. What is the probability that a beam particle will interact within the slab?

4. The condition for reactor criticality depends on the factor

$$k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a} > 1$$

where B satisfies the equation

$$\nabla^2\phi + B^2\phi = 0 .$$

a. Find an expression for the minimum macroscopic fission cross section Σ_f necessary for critical operation of an infinite, bare slab reactor of thickness a.

b. If the slab thickness increases to a very large value, what limiting value does k take on?

5. A slab-shaped neutron moderator with uniform macroscopic absorption cross-section Σ_a contains a planar source of neutrons at x=0. The slab is centered at x=0, has a finite thickness 2a, and the source has a strength S neutrons/(unit area – unit time). The diffusion equation is given as $D\nabla^2\phi - \Sigma_a\phi = -S$. Find the probability that a neutron emitted by the source will be absorbed within the slab. What happens to this probability as the slab thickness is reduced to zero?

6. In magnetic fusion the plasma pressure is balanced by a magnetic force which is proportional to the gradient of the magnetic energy density, $B^2 / 2\mu_0$.

a. If you have a cylindrical plasma of radius a, a pressure profile p=p(r) that is peaked at r=0, and an external magnetic field B, find the reduction of the magnetic field at r=0. What is the maximum pressure that could be confined for a given value of magnetic field?

b. Suppose you can now produce a plasma at this maximum pressure and the temperature of the plasma is a value T_0 that is high enough for fusion to occur. Expressing the density in terms of p_{max} and T_0, find the flux of neutrons that would then pass through the wall of the hypothetical fusion reactor. You may assume that the wall would be located at the boundary of the plasma.

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Solution

John Tiger Spring 2008

1) $W = \int_{\infty}^{r_n} \vec{F} \cdot d\underline{r}$

$F = \frac{q z' \partial\phi(r)}{\partial r}$

$W = \int_{\infty}^{r_n} q z \frac{\partial\phi(r')}{\partial r'} dr'$

$= q z \Delta\phi\Big|_{\infty}^{r_n} = \frac{q^2 z' z}{4\pi\epsilon_0 r_n}$

10pts

2. $W = \Delta M c^2$ where

$$\Delta M = M - m_N N - Z M_p$$

10 pts

3

$$dI = -I(x)\,dx\,[n_1\sigma_1 + n_2\sigma_2)$$

$\longrightarrow$

$$I(x) = I_0 e^{-x/\lambda}$$

$$\lambda = \frac{1}{n_1\sigma_1 + n_2\sigma_2}$$

a) $\boxed{I(t) = I_0 e^{-t/\lambda}\ ;\ \lambda \text{ above}}$

5 pts

b) probability of interacting in the slab

$$p = 1 - p'$$

p' ~ probability of leaving slab w/o interacting

$$p' = \frac{I(t)}{I_0} \rightarrow \boxed{p = 1 - \frac{I(t)}{I_0}}$$

5 pts

4a) from $k > 1$ condition

$$\Sigma_f > \frac{DB^2 + \Sigma_a}{\nu}$$

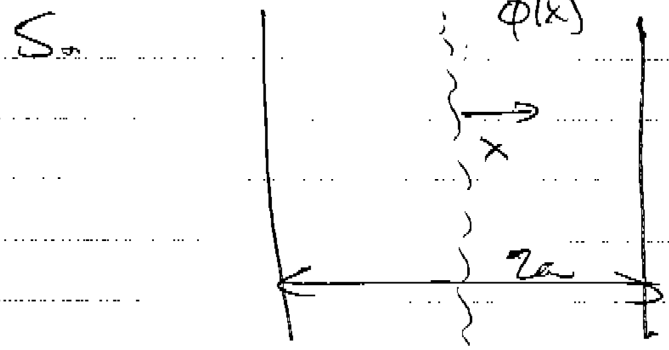
$$(\nabla^2 + B^2)\phi = 0 \longrightarrow$$

~~for~~ 1-D ∞ slab $B = \frac{\pi}{a}$

$$\therefore \boxed{\Sigma_f > \frac{D\left(\frac{\pi}{a}\right)^2 + \Sigma_a}{\nu}}$$ 5pts

b) $a \to \infty$ $B \to 0$ $\to$

$$\boxed{\Sigma_f > \frac{\Sigma_a}{\nu}}$$ 5pts



$\phi(x) = ?$

$D\nabla^2\phi - \Sigma_a\phi = 0 \quad x>0$

$\phi = A e^{-x/L}; \quad L^2 \equiv \frac{\Sigma_a}{D} \quad x>0$

$\phi = A e^{+x/L} \quad x<0$

$A = S/2$ from eval $\left.\phi(x)\right|_{\lim x\to 0} = \frac{1}{2}S$

$\therefore \phi = \frac{S}{2} e^{-x/L}; \quad L = \sqrt{\Sigma_a/D} \quad x>0$

φ(x=a) tells how many neutrons leaving RHS of the domain (φ(~~x=~~-a) gives same on LHS) (per unit area-time)

3pts

$\therefore p|_{\text{leaving RHS}} = \frac{\phi(x=a)}{S/2} = e^{-a/L}$

$\boxed{a\to 0 \quad p\to 0}$

∴ probability of absorption $\boxed{p = 1 - p_{leak} = 1 - e^{-a/L}}$

1pts

6)

a) ~~B(r=0)~~ =

$$p + \frac{B^2}{2\mu_0} = \text{const} = \frac{B_{ext}^2}{2\mu_0}$$

$$\therefore \quad B(r=0) = \left\{ \left[\frac{B_{ext}^2}{2\mu_0} - p(r=0) \right] (2\mu_0) \right\}^{1/2}$$

5 pts

$p_{max} = \frac{B_{ext}^2}{2\mu_0}$ will give $B(r=0) = 0$

b) $n_{max} = \frac{p_{max}}{T_0}$

evalat T_0

$P_{fusion} = \frac{n_{max}^2}{2} \langle \sigma v \rangle$ is the fusion power density fusion neutron flux, ϕ_n, will be given as

$$\phi_n = \frac{P_{fus} V \cdot f_n}{A_{wall}}$$

where f_n is the fraction of energy released in neutrons

$$[f_n = 0.8 \text{ for DT}]$$

$$V = \pi a^2 L \qquad \frac{V}{A_{wall}} = \frac{\pi a^2 \cancel{L}}{2\pi a \cancel{L}} = \frac{a}{2}$$

$$A_{wall} = 2\pi a L$$

$$\boxed{\therefore \phi_n = \frac{n_{max}^2}{2} \langle \sigma v \rangle \Big|_{T_0} f_n \frac{a}{2}}$$

SPB