Low-Re Hydrodynamics
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A - Transport and Fluids Fundamentals

- Continuum mechanics
- Reynolds transport theorem
- Mass conservation
- Newton's law, Stress tensor, Cauchy's equation of motion
- Newtonian flows and the Navier-Stokes equations
- Boundary conditions
- The Reynolds number; High vs. low Reynolds number limit

B - Stokes Flows: General

- Stokes (creeping-flow) equation: Equations for velocity, vorticity, pressure, stress
- Unidirectional flows, Couette, Poiseuille
- Unsteady shear flows and the kinematic viscosity
- Linear superposition, reversibility
- Energy balance, work, dissipation
- Uniqueness of solution
- Minimum dissipation theorem, and application to motion of a cube
- Reciprocal theorem, and application to cell motility
- Green's function: the Stokeslet
- Other singularities (point mass, force and source-dipoles, etc.)
- Faxen's law
- Integral representation of Stokes flows, and boundary integral method

C - Motion of bodies at low Reynolds numbers

- Force and Torques: Scalings
- General case: Mobilities and resistance tensor
- Bodies with symmetries, and velocity / rotation coupling
- Slender bodies
- Translation of a solid sphere (Stokes law): 3 ways to do the calculation
- Lamb's general solution
- Rotation of a solid sphere
- Translation of a spherical droplet and bubble
- Application: Diffusion of a sphere, Stokes-Einstein relationship
- Application: Sedimentation
- Bodies in shear flows

- Application: effective viscosity of suspension of non-Brownian spheres (Einstein's formula)
- Spheroids; Applications to cell motility
- Hydrodynamic interactions between particles: symmetries; far-field limit
- Wall effects: Lorentz image system
- Inertial effects: Oseen solution

D - Thin films

- The lubrication limit: fluid mechanics of thin films
- Spreading due to gravity
- Sedimentation of a sphere near a wall

References

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