280 B EXAM

An Euler Bernoulli Beam is described by

$$\ddot{\eta}_i + \omega_i^2 \eta_i = b_i^*(u+w) \qquad i = 1, \cdots N$$

$$y = \sum_i^N r_i \dot{\eta}_i$$

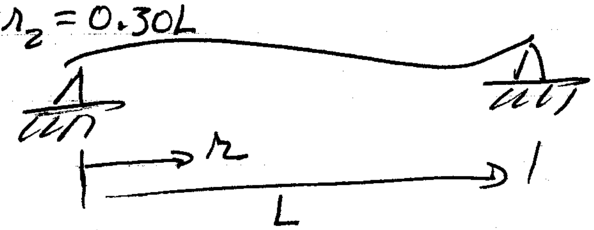
$$z = \sum_i^N r_i \dot{\eta}_i + v \qquad \begin{bmatrix} w \\ v \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix}\right)$$

$$W = I, \quad V = I$$

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

u_1 = force at location $r_1 = 0.45L$

u_2 = force at location $r = r_2 = 0.30L$



$$\omega_i = i^2$$

$$\psi_i(r_1) = \sqrt{\frac{2}{\rho L}} \sin\left(\frac{i\pi r_1}{L}\right) = \sin(0.45\pi i)$$

$$b_i^* = [\psi_i(r_1), \ \psi_i(r_2)], \quad r_i^* = b_i^*, \quad \psi_i(r_2) = \sin(0.3\pi i)$$

"<u>Control Problem</u>": Find a $2N^{th}$ order controller to

minimize $\mathcal{V} = \mathcal{E}_\infty u^* u$

subject to $\qquad \mathcal{E}_\infty y_i^2 < \epsilon_i^2$

(1) Find the smallest ϵ_i for which solutions to the "Control Problem" exist. Call this value $\underline{\epsilon}_i$.

(2) Solve the "Control Problem" for $\epsilon_i = \frac{1}{2}\underline{\epsilon}_i$.

Solve Problems (1), (2) two ways.

(i) using iterating Riccati solutions

(ii) using LMIs.

(3) Find the smallest $\mu_i = \underline{\mu}_i$ for which solutions exist for this problem

$$\text{minimize} \quad \mathcal{V} = \mathcal{E}_\infty y^T y$$
$$\text{subject to:} \quad \mathcal{E}_\infty u_i^2 \le \mu_i^2$$

(4) Set $\mu_i = \frac{1}{2}\underline{\mu}_i$ and solve problem (3) for a $2N^{th}$ order controller.

(5) Find a <u>state feedback</u> controller to solve (1), (2), (3), (4).

Discuss issues that were educational or puzzling during your solution of this exam.