MAE231B
Mid-term Exam
(Closed Book & Notes)

Date: February 20, 2007

Time: 3:35 to 4:45pm

Name: ______________________________

Major Department: _________________________

Note: In all relevant cases below:

$\mathbf{e}_i$, $i = 1, 2, 3$, is an orthogonal set of unit vectors defining a rectangular Cartesian coordinate system in 3-D.

Problem 1

Consider the following vector-valued function:

$$\mathbf{u}(x_1,\ x) = x_1^3 \sin(x_2 \pi)\, \mathbf{e}_1 + x_2^2 \sin(x_1 \pi)\, \mathbf{e}_2 .$$

(1.5 Points) Calculate $\nabla \bullet \mathbf{u}$, $\nabla \times \mathbf{u}$, and $\nabla \otimes \mathbf{u}$.

Problem 2

Consider a solid occupying a region R bounded by a regular surface ∂R. The solid is subjected to body forces, $\mathbf{f}^o$, and surface tractions, $\mathbf{t}^o$, on a part, ∂R_T, of its surface. On the remaining part, ∂R_U, of ∂R, surface displacements are prescribed to be $\mathbf{u} = \mathbf{u}^o$.

(a) (2 Points) Write down all the conditions that a stress field, $\boldsymbol{\sigma}(\mathbf{x})$, must satisfy in order to be *statically admissible*.

(b) (1 Point) Write down all the conditions that a displacement field, $\mathbf{u}(\mathbf{x})$, must satisfy in order to be *kinematically admissible*.

(c) (1.5 Points) Show that if $\mathbf{u}(\mathbf{x})$ is any kinematically admissible displacement field, and $\boldsymbol{\sigma}(\mathbf{x})$ is any statically admissible stress field, then

$$\int_R \boldsymbol{\sigma} : \boldsymbol{\varepsilon}\, dV = \int_R \mathbf{f}^o \bullet \mathbf{u}\, dV + \int_{\partial R_T} \mathbf{t}^o \bullet \mathbf{u}\, dS ,$$

where $\boldsymbol{\varepsilon}$ is the strain field corresponding to $\mathbf{u}$.

Note: To receive credit you must start with the left-hand expression of the above equation, and systematically deduce the right-hand expression.

(d) (1.5 Points) What conditions the kinematically admissible displacement field, $\mathbf{u}(\mathbf{x})$, must satisfy if it is also statically admissible? Give all the equations and also explain in words.

Problem 3

The strain tensor at a fixed point in a deformed solid has the following components:

$$[\varepsilon_{ij}] = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \times 10^{-3}.$$

(a) (2 Points) Find the principal strains and their directions.

(b) (2 Points) Find the principal shear strains and their directions.

(c) (1 Point) Find the strain of an element in the direction defined by the unit vector $\mathbf{m} = (2\,\mathbf{e}_1 - \mathbf{e}_2 + \mathbf{e}_3)/\sqrt{6}$.

(d) (1.5 Points) Find the strain of the two orthogonal elements, one in the direction defined by the unit vector $\mathbf{m}$, and the other by $\mathbf{n} = (2\,\mathbf{e}_1 + 3\,\mathbf{e}_2 - \mathbf{e}_3)/\sqrt{14}$. Show that $\mathbf{m}$ and $\mathbf{n}$ are mutually orthogonal unit vectors.

(e) (1 Point) Write down the Hooke's Law for an isotropic solid and define all terms.

Note: To receive full credit, you must show all your calculations.