Soln. MT 6.9

(a) For this planar flow the stream function satisfies:

$$u = \frac{\partial \psi}{\partial y} = U \sin\frac{\pi x}{L} \cos\frac{\pi y}{L},$$

$$v = -\frac{\partial \psi}{\partial x} = U \sin\frac{\pi y}{L} \cos\frac{\pi x}{L}$$

which gives $\quad \psi = \frac{LU}{\pi} \sin\frac{\pi x}{L} \sin\frac{\pi y}{L} \quad$ (for $\psi = 0$ at $y=0$)

Expansion:

$$\psi = \psi_0(x) + \psi_1(x)\, y + \psi_2(x)\, y^2 + \dots = LU\psi^{\#}$$

$$\psi_0 = 0, \quad \psi_1^{\#} = \frac{\psi_1}{U}(x) = \sin \pi x^{\#}, \quad \text{etc.},$$

$\therefore$ $n = 1$ (as appropriate to gas liquid interface)

$\therefore$ From JDG MT 15-6, eq. (12), ($x \to x^{\#}$, $r_0 \equiv 1$)

$$\overline{Sh} = \frac{L\bar{k}_L}{\mathcal{D}} = \frac{\left[4 \int_0^1 \psi_1^{\#}(x^{\#})\, dx^{\#}\right]^{1/2}}{\Gamma(1/2) \int_0^1 dx^{\#}} (Pe)^{1/2}$$

$$= \frac{2}{\sqrt{\pi}} \left[\int_0^1 \sin \pi x \, dx\right]^{\frac{1}{2}} (Pe)^{1/2} = \frac{2^{3/2}}{\pi} (Pe)^{1/2}$$

$$= 0.9003\,(Pe)^{1/2}$$

$(Pe \equiv UL/\mathcal{D})$

$$= 1.273\left(\frac{Pe}{2}\right)^{1/2}$$

(b) The equivalent result of F & P is

$$\overline{Sh} = 1.46 \left(\frac{Pe}{2}\right)^{1/2} \qquad \left(\frac{Pe}{2} = \frac{1}{2} UL/\mathcal{D} = \frac{\sqrt{\bar{u}'^2}\,\lambda}{\mathcal{D}}\right)$$

The result of (a) actually compares better with data. (See Fig 11 of F & P)