a.) $r = \sqrt{\rho^2 + x^2} = x\sqrt{1+\rho^2/x^2} \sim x\left[1 + \rho^2/2x^2 + \cdots\right]$

for $x \gg \rho$,

$\therefore$

$$\frac{Q}{4\pi D r} e^{-\frac{U}{2D}(r-x)} \sim \frac{Q}{4\pi D x} e^{-\frac{U\rho^2}{4Dx}} \quad \text{for} \quad x \gg \rho$$

b.) Both $G(x,y,z\pm z_0)$ are solutions to convection equation (by superposition). Also

$$\left[\frac{\partial G}{\partial z}(x,y,z\pm z_0)\right]_{z=0} = \left[\frac{\partial r_\pm}{\partial z}\frac{\partial G}{\partial r_\pm}(x,y,z\pm z_0)\right]_{z=0}$$

$$= \left[\left(\frac{z\pm z_0}{r_\pm}\right)\frac{\partial G}{\partial r_\pm}(x,y,\pm z_0)\right]_{z=0}$$

$$= \frac{\pm z_0}{r_0}\left[\frac{\partial G}{\partial r}(r,x)\right]_{r=r_0}$$

where $r_\pm = \sqrt{\rho^2 + (z\pm z_0)^2}$, $r_0 = \sqrt{\rho^2 + z_0^2}$

and $G(r,x) \equiv G(x,y,z)$ given by (1).

Therefore

$$\left[\frac{\partial}{\partial z}\left\{G(x,y,z+z_0) + G(x,y,z-z_0)\right\}\right]_{z=0}$$

$$= \frac{z_0}{r_0}\left(\frac{\partial G}{\partial r}\right)_{r_0} - \frac{z_0}{r_0}\left(\frac{\partial G}{\partial r}\right)_{r_0} \equiv 0$$

So that $G(x,y,z+z_0) + G(x,y,z-z_0)$ satisfies no flux condition at $z=0$.

(c) $$C_0 = (C)_{z=0} = G(x, y, z_0) + G(x, y, -z_0)$$

$$\doteq \frac{Q}{2\pi D x} e^{-\frac{U\rho_0^2}{4Dx}}, \text{ where } \rho_0^2 = y^2 + z_0^2$$

$$0 = \left(\frac{\partial C_0}{\partial x}\right) = -\frac{1}{x^2}\left(\frac{\partial C}{\partial(1/x)}\right) = -\frac{Q}{2\pi D x^2}\frac{\partial}{\partial(1/x)}\left[\frac{1}{x}e^{-a/x}\right]$$

$$= -\frac{Q}{2\pi D x^2}\left[e^{-a/x} - a\left(\tfrac{1}{x}\right)e^{-a/x}\right]$$

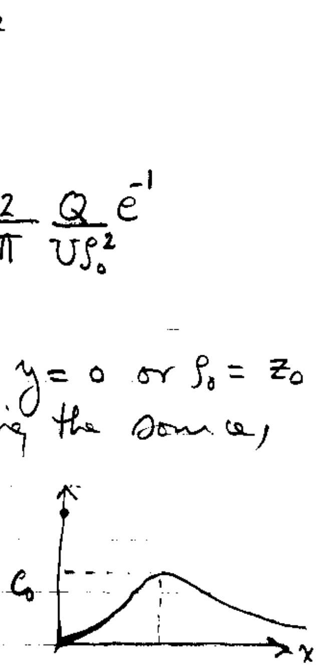
This occurs at $x = a \equiv \dfrac{U\rho_0^2}{4D}$

where

$$C_0 = \frac{Q}{2\pi D \frac{U\rho_0^2}{4D}} e^{-1} = \frac{2}{\pi}\frac{Q}{U\rho_0^2}e^{-1}$$

Largest value of this occurs at $y = 0$ or $\rho_0 = z_0$ along symmetry plane containing the source, where

$$(C_0)_{max.} = C_m = \frac{2e^{-1}}{\pi}\frac{Q}{U z_0^2}$$



at $x = x_m = \dfrac{U z_0^2}{4D}$

Then, at $y=0$, $$\frac{C_0}{C_m} = \left(\frac{x_m}{x}\right)e^{1-(x_m/x)}$$