Solution MT 10.3

a) Recall $k_L^\circ = 2\sqrt{\frac{\mathcal{D}}{\pi t}}$ t = total contact time

$$\therefore \quad t = \frac{4}{\pi}\frac{\mathcal{D}}{(k_L^\circ)^2}$$

and

$$u = kt = \frac{4}{\pi}\frac{k\mathcal{D}}{(k_L^\circ)^2} \equiv \frac{4}{\pi}\gamma_0$$

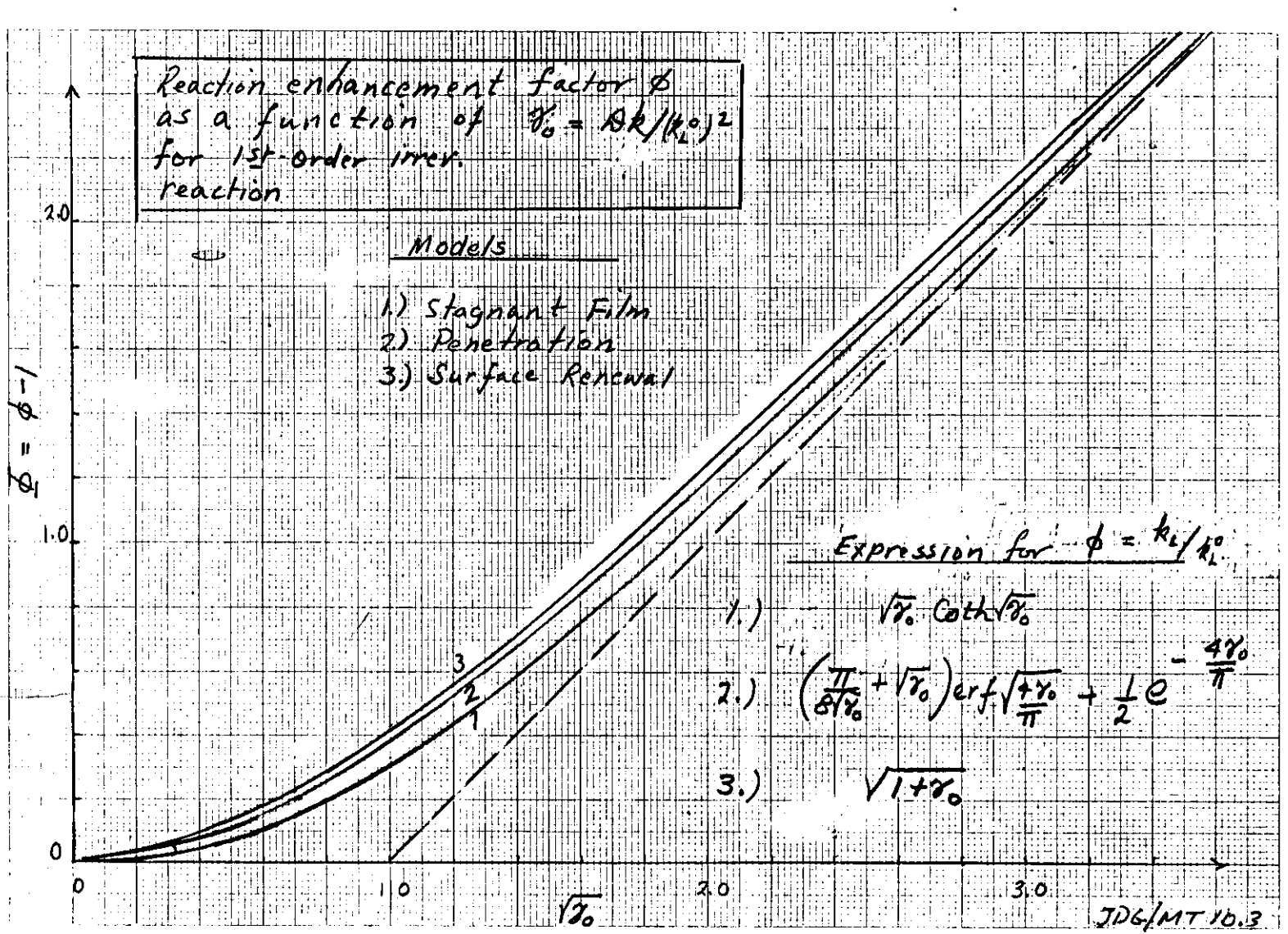
Hence

$$\varphi = \frac{\pi}{4\sqrt{\gamma_0}}\left[\left(\frac{1}{2} + \frac{4\gamma_0}{\pi}\right)\operatorname{erf}\sqrt{\frac{4}{\pi}\gamma_0} + \frac{2}{\pi}\sqrt{\gamma_0}\, e^{-\frac{4\gamma_0}{\pi}}\right]$$

b) Calculations:

		φ		
γ_0	$\sqrt{\gamma_0}$	Penetration Model	Film model	Surface Renewal
0.1	.316	1.042	1.033	1.049
0.5	.707	1.200	1.161	1.225
1.0	1.0	1.378	1.313	1.414
5.0	2.24	2.41	2.288	2.449
10	3.16	3.286	3.174	3.317

See plot.


Reaction enhancement factor Ø as a function of $\gamma_0 = Dk/(k_L^0)^2$ for 1st-order irrev. reaction
Models
1.) Stagnant Film
2.) Penetration
3.) Surface Renewal
Expression for $\phi = k_L/k_L^0$
1.) $\sqrt{\gamma_0}\coth\sqrt{\gamma_0}$
2.) $\left(\frac{\pi}{8\gamma_0}+\sqrt{\gamma_0}\right)\operatorname{erf}\sqrt{\frac{\pi}{4\gamma_0}}+\frac{1}{2}e^{-\frac{4\gamma_0}{\pi}}$
3.) $\sqrt{1+\gamma_0}$
$\phi - 1 = \phi_1$
2.0
1.0
0
1.0
2.0
3.0
$\sqrt{\gamma_0}$
3
2
1
JDG/MT 10.3

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