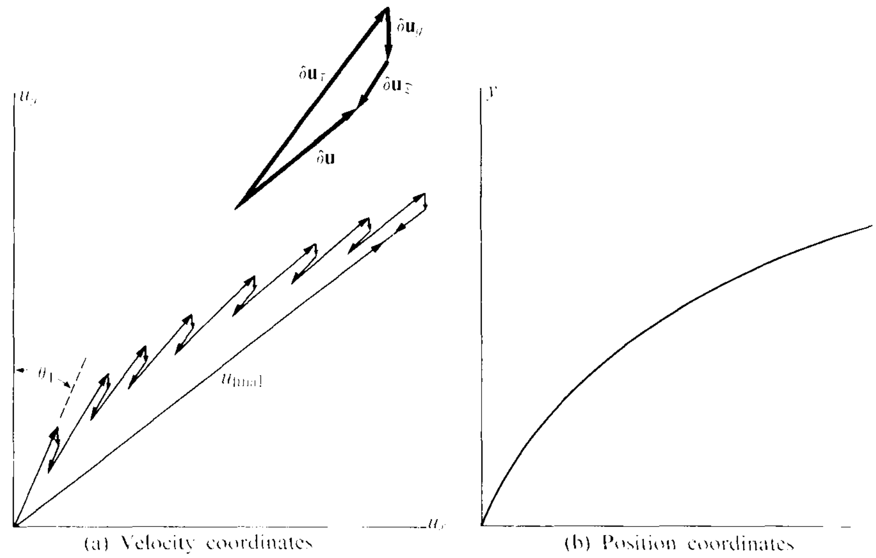

FIGURE 10.5 Approximate calculation of the trajectory of a vehicle undergoing a gravity from launch at angle θ_1.

$\delta \mathbf{u}_{\mathscr{D}}$ due to drag, parallel and opposite to $\delta \mathbf{u}_{f}$; and $\delta \mathbf{u}_{g}$ due to gravity, paral **g**. Each of these terms can be calculated from the velocity at the beginning Of course, the accuracy of this procedure depends on the size of the time vals δt compared with t_b.

Having obtained the solution of **u** as a function of time, one can easily pl trajectory, Fig. 10.5(b) from the velocity diagram. That is, $\delta \mathbf{y} = \mathbf{u}_i\, \delta t$, δx - for each δt. This figure is intended only to indicate the nature of the com problem and to point out that for our purposes we are interested simply summation of the magnitudes of the $\delta \mathbf{u}_{f}$ terms. Clearly, this is

$$\sum |\delta \mathbf{u}_{f}| = |\mathbf{u}_{eq}| \ln \mathscr{R} .$$

Thus, having properly calculated the actual trajectory, the simple integrat Eq. (10.9), as given by Eq. (10.11), adequately describes the propulsion re ments for a given mission even in the presence of gravity and drag.

Single-Stage Sounding Rocket

As a simple example, consider the height to which a single-stage rock rise if we neglect drag and assume that the effective exhaust velocity is con For vertical flight the altitude attained at burnout, h_b, is

$$h_b = \int_0^{t_b} u\, dt ,$$

where u is given by

$$u = -u_e \ln \frac{\mathcal{M}}{\mathcal{M}_0} - g_e t .$$

If the rate of fuel consumption is constant, the mass varies with time as

$$\mathcal{M}(t) = \mathcal{M}_0 - (\mathcal{M}_0 - \mathcal{M}_b)t/t_b .$$

Then

$$u = -u_e \ln\left[1 - \left(1 - \frac{1}{\mathcal{R}}\right)\frac{t}{t_b}\right] - g_e t , \tag{10.15}$$

and

$$h_b = -u_e t_b \frac{\ln \mathcal{R}}{\mathcal{R} - 1} + u_e t_b - \frac{1}{2} g_e t_b^2 . \tag{10.16}$$

Equating the kinetic energy of the mass at burnout, $\mathcal{M}_b$, with its change of potential energy between that point and the maximum height, h_{max}, we obtain

$$\mathcal{M}_b \frac{u_b^2}{2} = \mathcal{M}_b g_e (h_{max} - h_b) ,$$

and thus

$$h_{max} = h_b + \frac{u_b^2}{2g_e} . \tag{10.17}$$

Finally,

$$h_{max} = \frac{u_e^2 (\ln \mathcal{R})^2}{2g_e} - u_e t_b \left(\frac{\mathcal{R}}{\mathcal{R} - 1} \ln \mathcal{R} - 1\right) . \tag{10.18}$$

Burning Time

The result we have just obtained points out the desirability of reducing the burning time as much as possible while accelerating against a gravity field. Physically, short burning times reduce the energy consumed in simply lifting the propellant. Very short burning times, however, imply not only very high accelerations, which may impose severe stresses on the structure and instruments, but also exceedingly high propellant flow rates. The size of the machinery necessary to handle large flows is a limiting factor. In addition, atmospheric drag imposes a penalty if the vehicle is accelerated too quickly within the earth's atmosphere. Burning times for existing high-thrust rockets are usually in the range of 30 to 200 seconds.

In the absence of gravitation or drag, burning time has no influence on stage velocity increment, as we may see from Eq. (10.11). In this case the velocity of the vehicle is a function only of the fraction of propellant expended and not of the time consumed in acceleration.