Homework 2
MAE 118C

Problems 7, 9, 10, 13, 16, 17, 25, 27, 30, 40, 44 from Chapter 3, Lamarsh & Baratta

7.

A beam of neutrons is incident from the left on a target that extends from x=0 to x=a. Derive an expression for the probability that a neutron in the beam will have its first collision in the second half of the target that is in the region a/2 < x < a.

$$I(x) = I_0 \cdot e^{-\Sigma_t \cdot x}$$

$$p(x)\,dx = \Sigma_t \cdot e^{-\Sigma_t \cdot x} \cdot dx$$

$$p(x) = \int_{\frac{a}{2}}^{a} e^{-\Sigma_t \cdot x}\,dx \qquad p(x) = \frac{-1}{\Sigma_t} \cdot \left(e^{-a \cdot \Sigma_t} - e^{\frac{-a \cdot \Sigma_t}{2}} \right)$$

9.

What is the probability that a neutron can move one mean free path without intereacting in a medium?

$$p(x)\,dx = \Sigma_t \cdot e^{-\Sigma_t \cdot x} \cdot dx$$

$$p(x) = \Sigma_t \cdot \int_0^{\lambda} e^{-\Sigma_t \cdot x}\,dx$$

$$p = -1 \cdot \left(e^{-1} - e^{0} \right)$$

$$p = 1 - e^{-1} = .63$$

10.

collision density $F = I \cdot N \cdot \sigma_t$

collisions per second = $F \cdot A \cdot X$

σ_a >> σ_s so all collisions are absorbed

rate of neutrons absorbed in target = $\sigma_a \cdot \phi_o \cdot N \cdot A \cdot X$

13.

$$\Sigma_a = \sum_i \left(N_i \cdot \sigma_i\right)$$

$$\Sigma_a = \sum_i \left(N_i \cdot \sigma_i \cdot \frac{N}{N}\right)$$

$$\Sigma_a = \sum_i \left(f_i \cdot \sigma_i \cdot N\right)$$

$$\Sigma_a = \sum_i \left(f_i \cdot \Sigma_{ai}\right)$$

16. $eV := 1.602 \cdot 10^{-19} \cdot J$ $\quad MeV := 10^6 \cdot eV$

$\Gamma_n := 1.52 \cdot 10^{-3} eV$ $\quad E_r := 6.67 \cdot eV$ $\quad h := 6.626 \cdot 10^{-34} \cdot \frac{m^2 \cdot kg}{s}$

$\Gamma_\gamma := 26 \cdot 10^{-3} eV$ $\quad \Gamma_T := \Gamma_n + \Gamma_\gamma$

$g_1 := 1$ $\quad \gamma_r := \frac{h \cdot c}{E_r} = 0.186 \cdot \mu m$

$N_1 := 200$ $\quad i := 0 .. N_1$ $\quad n := 0 .. N_1$

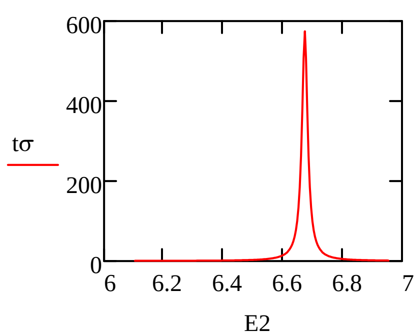
$$E_i := \frac{1 \cdot i \cdot eV}{N_1} + 6eV$$

$$\sigma_i := \frac{\gamma_r^{\,2} \cdot g_1}{4 \cdot \pi} \cdot \frac{\Gamma_n \cdot \Gamma_\gamma}{\left(E_i - E_r\right)^2 + \frac{\Gamma_T^{\,2}}{4}}$$

Scaling factors so the plot looks right.

$$E2 := \frac{E}{2} \cdot 1.062 \cdot 10^{19} + 1J$$

$$t\sigma_i := \sigma_i \cdot 10^{18}$$



The vertical scale has been rescaled by 10^18 and the horizontal scale is in the units of eV.

17.

$$\sigma_i = \frac{\gamma_r^2 \cdot g_1}{4\cdot\pi} \cdot \frac{\Gamma_n \cdot \Gamma_\gamma}{\left(E_i - E_r\right)^2 + \frac{\Gamma_T^2}{4}}$$

Max at Ei=Er

$$\sigma_{max} = \frac{\gamma_r^2 \cdot g_1}{\pi} \cdot \frac{\Gamma_n \cdot \Gamma_\gamma}{\Gamma_T^2}$$

$$\frac{\sigma_{max}}{2} = \frac{\gamma_r^2 \cdot g_1}{2\cdot\pi} \cdot \frac{\Gamma_n \cdot \Gamma_\gamma}{\Gamma_T^2} = \frac{\gamma_r^2 \cdot g_1}{4\cdot\pi} \cdot \frac{\Gamma_n \cdot \Gamma_\gamma}{\left(E_i - E_r\right)^2 + \frac{\Gamma_T^2}{4}}$$

Solve for (Ei-Er) which is half the width:

$$\frac{1}{\Gamma_T^2} = \frac{1}{\left(E_i - E_r\right)^2 - \frac{\Gamma_T}{4}}$$

$$\left(E_i - E_r\right)^2 = \frac{\Gamma_T^2}{2}$$

$$\left(E_i - E_r\right) = \frac{\Gamma_T}{2}$$

So the full width at half its hieght is equal to Γ_T Im using gamma sub T while the book just uses gamma.

25.

a) Head-on collision theta = pi

$A_O := 16$ $\quad E_{initial} := 2\cdot MeV$ $\quad k := 1.38\cdot 10^{-23}\cdot\frac{J}{K}$

$$\alpha := \left(\frac{A_O - 1}{A_O + 1}\right)^2 = 0.779$$

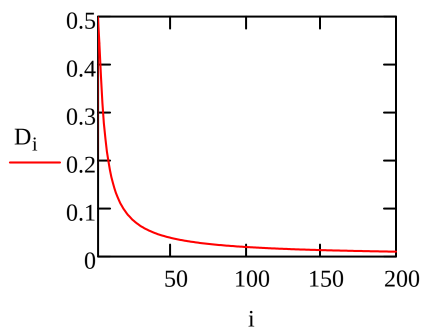
$$E_{nuetron} := \alpha\cdot E_{initial} = 1.557\cdot MeV$$

$$E_{nucleus} := E_{initial} - E_{nuetron} = 0.443\cdot MeV$$

b) The binding energy per nucleon for O is larger than this number so it is very unlikely the nucleus will split.

27. $N_1 := 200$ $\quad i := 0 .. N_1$ $\quad M := 1 .. N_1$

$D_1 = \frac{\Delta E}{E}$ $\quad D_i := \frac{1}{2}\cdot\left[1 - \left(\frac{i-1}{i+1}\right)^2\right]$



$E_g := 1.71\text{MeV}$

30.

$E_{int} := 2\cdot\text{MeV}$ $\quad A_H := 1$ $\quad \xi_H := 1$ $\quad \ln\left(\frac{E_{int}}{E_g}\right) = 0.157$

$E_{fin} := 1\text{eV}$ $\quad A_C := 12$

$$\xi_C := 1 - \frac{(A_C - 1)^2}{2\cdot A_C}\cdot\ln\left(\frac{A_C + 1}{A_C - 1}\right) = 0.158$$

a) $\ln\left(\frac{E_{int}}{E_{fin}}\right)\cdot\frac{1}{\xi_H} = 14.509$ collisions required for hydrogen moderator

b) $\ln\left(\frac{E_{int}}{E_{fin}}\right)\cdot\frac{1}{\xi_C} = 91.961$ collisions required for graphite moderator

40.

Fission Rate = $2.70\cdot 10^{21}\cdot P = \frac{\text{fissions}}{\text{day}}$

Creations of ^{131}I per unit time= $R = 8.91\cdot 10^{24}\cdot 0.029 = 2.58\cdot 10^{23}\cdot\frac{\text{creations}}{\text{day}}$

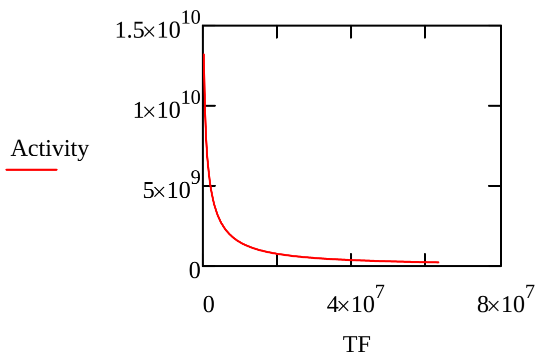
$\alpha_{equilibrium} = R = 2.58\cdot 10^{23}\cdot\frac{\text{decays}}{\text{day}} = 8.08\cdot 10^{7}\cdot\text{Ci}$

44.

$P := 250 \cdot kW \qquad TF_i := \frac{(i+1) \cdot 52 \cdot 2 \cdot 7}{N_1} \cdot day$

$T_1 := 52 \cdot 5 \cdot 8 \cdot hr = 86.667 \cdot day$

$Activity_i := 1.4 \cdot 10^6 \cdot P \cdot \left[\left(TF_i\right)^{-.2} - \left(TF_i + T_1\right)^{-.2} \right]$



Complete the derivations of equation 2.55 in Lamarsh&Barratt chapter 2

Show

$$\bar{E} = \frac{3}{2} \cdot k \cdot T$$

$$\bar{E} = \frac{1}{N} \cdot \int_0^{\infty} N(E) \cdot E \, dE \qquad N(E) = \frac{2 \cdot \pi \cdot N}{(\pi \cdot k \cdot T)^{\frac{3}{2}}} \cdot E^{\frac{1}{2}} \cdot e^{\frac{-E}{kT}}$$

$$\bar{E} = \frac{1}{N} \cdot \int_0^{\infty} \frac{2 \cdot \pi \cdot N}{(\pi \cdot k \cdot T)^{\frac{3}{2}}} \cdot E^{\frac{3}{2}} \cdot e^{\frac{-E}{kT}} \, dE$$

$$\bar{E} = \frac{2 \cdot \pi}{(\pi \cdot k \cdot T)^{\frac{3}{2}}} \cdot \int_0^{\infty} E^{\frac{3}{2}} \cdot e^{\frac{-E}{kT}} \, dE$$

then from integral tables

$$\bar{E} = \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot\frac{\Gamma\left(\frac{5}{2}\right)}{\left(\frac{1}{kT}\right)^{\frac{5}{2}}}$$

$$\Gamma\left(\frac{5}{2}\right) = \sqrt{\pi}\cdot\frac{3}{4}$$

$$\bar{E} = \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot\frac{\sqrt{\pi}\cdot\frac{3}{4}}{\left(\frac{1}{kT}\right)^{\frac{5}{2}}}$$

$$\bar{E} = \frac{3}{2}\cdot kT$$

If the energy distribution function is given by equation 2.52 and a system has a temperature T, what fraction of the particles have a kinetic energy E>kT?

$$N(E) = \frac{2\cdot\pi\cdot N}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}$$

$$P = \int_{kT}^{\infty} \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}\, dE$$

$$P = \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot\int_{kT}^{\infty} E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}\, dE$$

Let $x = \frac{E}{kT}$ $dx = \frac{dE}{kT}$

$$P = \int_{1}^{\infty} \frac{2}{\sqrt{\pi}}\cdot x^{\frac{1}{2}}\cdot e^{-x}\, dx = 0.572$$

Full credit for setting up the integral, solving it is bonus.

Refering to equation 3.28 in L&B, show that the minimum value of kinetic energy for an elastically scattered neutron occurs when the scattering angle equals 180 degrees.

$$E_{prime} = \frac{E}{(A+1)^2}\cdot\left(\cos\theta + \sqrt{A^2 - \sin^2\theta}\right)^2$$

$$\frac{d}{d\theta}E_{Prime} = \frac{E\cdot 2}{(A+1)^2}\cdot\left(\cos\theta + \sqrt{A^2 - \sin^2\theta}\right)\cdot\left[-\sin\theta + 2\cdot\left(A^2 - \sin^2\theta\right)\right]\cdot(-2\cdot\sin\theta\cos\theta)$$

Maxima or minima where derivitive equals zero which happens when theta = 0 or π, theta = 0 is clearly a maxima, theta = π is a minima.

$$\theta = \pi$$

$$E_{prime} = \frac{E}{(A+1)^2}\cdot\left(\cos\pi + \sqrt{A^2 - \sin^2\pi}\right)^2$$

$$E_{prime} = \frac{E}{(A+1)^2}\cdot\left(-1 + \sqrt{A^2 - 0}\right)^2$$

$$E_{prime} = \frac{E}{(A+1)^2}\cdot(-1 + A)^2$$

$$E_{prime} = \frac{E}{(A+1)^2}\cdot(A - 1)^2$$

$$E_{prime} = \alpha\cdot E \qquad \alpha \le 1$$

Suppose that the macroscopic absorption cross section was a constant for a particular type of material. Find the total absorption rate, F, of such an absorber that is exposed to a thermal population of neutrons using equation 3.38.

$$F_a = \int n(E)\cdot\nu(E)\cdot\Sigma_a(E)\,dE$$

$$\nu(E) = \sqrt{\frac{2}{m}}\cdot E^{\frac{1}{2}}$$

$$F_a = \Sigma_a\cdot\int n(E)\cdot\nu(E)\,dE$$

$$n(E) = \frac{2\cdot\pi\cdot N}{(\pi\cdot k\cdot T)^{\frac{3}{2}}}\cdot E^{\frac{1}{2}}\cdot e^{\frac{-E}{kT}}$$

$$F_a = \Sigma_a \cdot \int \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}} \cdot E^{\frac{1}{2}} \cdot e^{\frac{-E}{kT}} \cdot \sqrt{\frac{2}{m}} \cdot E^{\frac{1}{2}} \, dE$$

$$F_a = \frac{2\cdot\pi}{(\pi\cdot k\cdot T)^{\frac{3}{2}}} \cdot \sqrt{\frac{2}{m}} \cdot \Sigma_a \cdot \int E\cdot e^{\frac{-E}{kT}} \, dE$$

$$F_a = \sqrt{\frac{8kT}{m\,\pi}} \cdot \Sigma_a$$