Solution to HW2

1.

(a)

For the *nth* year:

$R_C = 6*(1+r_1)^n * 5000*(1+r_2)^n * E(1-r_3)^n * C*(1-r_C)^n$, here $r_C = 0$ for the fixed carbon intensity

(b)

To keep the rate of CO_2 emission, i.e. $R_C = const$.

$$\Rightarrow (1+r_1)(1+r_2)(1-r_3) = 1$$

$$\Rightarrow r_3 = 1 - \frac{1}{(1+r_1)(1+r_2)}$$

(c)

To keep C emission rate constant

$$\Rightarrow (1+r_1)(1+r_2)(1-r_3)(1-r_C) - 1 = 0$$

$$\Rightarrow r_3 = 1 - \frac{1}{(1+r_1)(1+r_2)(1-r_C)}$$

$$= 1 - \frac{1}{(1+0.01)(1+0.03)(1-0.02)}$$

$$= 0.019 = 1.9\%$$

2.

Energy flux in: $I_{in} = I_0 - RI_0$

Energy flux out: $I_{out} = bE$

Energy balance: $I_{in} = I_{out}$

$$I_0 - RI_0 = bE$$

$$\Rightarrow E = \frac{I_0 - RI_0}{b}$$

Since it's blackbody, $E = \sigma T^4$

$$\Rightarrow T = \sqrt[4]{\frac{I_0 - RI_0}{b\sigma}}$$

3.

$$\int_0^d dI(x) = -\int_0^d s_1 N_{gg1} I(x)dx + s_2 N_{gg2} I(x)dx$$

$$\Rightarrow \int_0^d \frac{dI(x)}{I(x)} = -\int_0^d (s_1 N_{gg1} + s_2 N_{gg2})dx$$

$$\Rightarrow \ln(I(x))\Big|_0^d = -(s_1 N_{gg1} + s_2 N_{gg2})d$$

$$\Rightarrow \ln\frac{I(d)}{I(0)} = -(s_1 N_{gg1} + s_2 N_{gg2})d$$

$$\Rightarrow \frac{I(d)}{I(0)} = \exp(-(s_1 N_{gg1} + s_2 N_{gg2})d)$$

$$\because \frac{I(d)}{I(0)} = \beta$$

$$\therefore \beta = \exp(-(s_1 N_{gg1} + s_2 N_{gg2})d)$$

4.

$E_1 = A_1 + (1-\beta_{IR})E_2$

$E_2 = A_2 + f_{earth}E_1$

From the notes, we know

$A_1 = [1-\alpha_1 - \beta_{vis} + \beta_{vis}\alpha_2(1-\beta_{vis})]I$

$A_2 = \beta_{vis}I - \alpha_2\beta_{vis}I$

where,

$I \sim 344\ \text{W/m}^2$

$\alpha_1 \sim 0.33$

$\beta_{vis} \sim 0.49$

$\alpha_2 \sim 0.04$

$f_{earth} \sim 0.64$

$\Rightarrow A_1 \approx 65.36$

$A_2 \approx 161.82$

$$\therefore \int E_1 = 65.36 + (1-\beta_{IR})E_2 \quad (1)$$
$$E_2 = 161.82 + 0.64E_1 \quad (2)$$

plug (2) into (1)

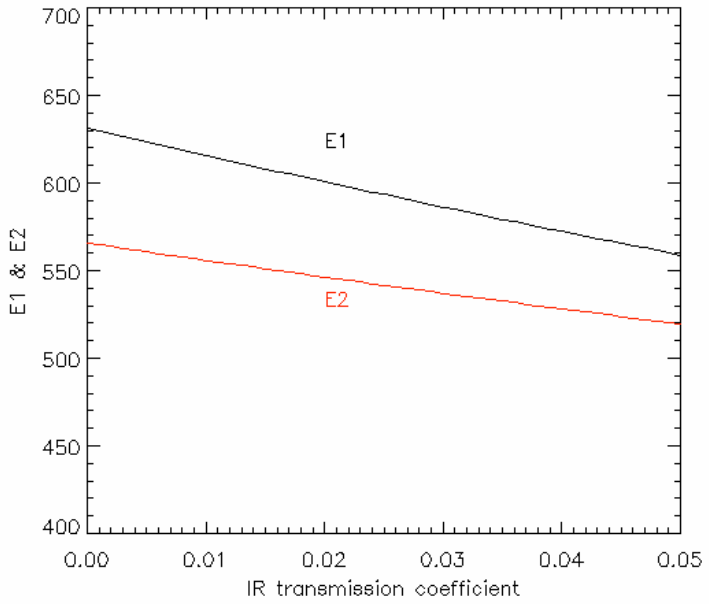
$$\Rightarrow E_1 = \frac{227.18 - 161.82\beta_{IR}}{0.36 + 0.64\beta_{IR}}$$

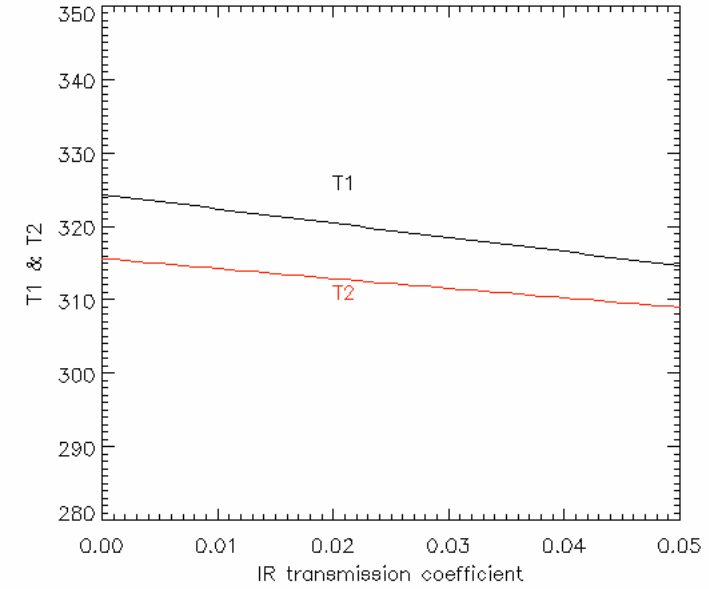
$E_1 = \sigma T_1^4$

$E_2 = \sigma T_2^4$

$where, \sigma = 5.7\times10^{-8}\ \text{W/m}^2K^4$

$$\Rightarrow T_1 = \sqrt[4]{\frac{E_1}{\sigma}}, T_2 = \sqrt[4]{\frac{E_2}{\sigma}}$$





5.

From notes we know,

$\beta_{pre} = \exp(-sn_{gg}d)$

when n_{gg} is doubled

$\beta_{cur} = \exp(-2sn_{gg}d) = \beta_{pre}^2$

assuming CO2 is the only green house gas in the atmosphere

6.

From IEA report,

1971~2002: the CO2 growth rate is ~1.75%

energy growth rate is ~2%

2002~2030: the CO2 growth rate is ~1.8%

energy growth rate is ~1.75%

Most increase in CO2 emissions in 2002~2030 is from developing countries which may not have efficient way to reduce CO2 emissions.

From the notes, current CO2 value is ~380ppm

$$C_{\mathrm{CO2\_current}}(1+r_{CO2})^{n} = C_{CO2\_\max}$$

$$\Rightarrow 380(1+0.018)^{n} = 500$$

$$\Rightarrow n\ln 1.018 = \ln(\frac{500}{380})$$

$$\Rightarrow n \approx 15$$