HW8

Problem 1 A normal shock stands ahead of engine inlet of a supersonic aircraft flying at Mach 1.6 at 10 km (standard atmosphere; Appendix A). The internal pressure loss is 0.97. Find

a) The external pressure loss.
b) The total pressure loss.
c) The diffuser isentropic efficiency.

Solution:

a)

$$\frac{P_{t2}}{P_{t1}}=\left[\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]^{\gamma/(\gamma-1)}\left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{1/(\gamma-1)}$$

$$\frac{P_{t2}}{P_{t1}}=\left[\frac{(1.4+1)\cdot(1.6)^2}{(1.4-1)\cdot(1.6)^2+2}\right]^{1.4/(1.4-1)}\left[\frac{1.4+1}{2\cdot1.4\cdot(1.6)^2-(1.4-1)}\right]^{1/(1.4-1)}$$

$$\frac{P_{t2}}{P_{t1}}=\left[\frac{(2.4)\cdot(1.6)^2}{(0.4)\cdot(1.6)^2+2}\right]^{7/2}\left[\frac{2.4}{2.8\cdot(1.6)^2-(1.4-1)}\right]^{1/(0.4)}$$

$$\frac{P_{t2}}{P_{t1}}=0.8952=\pi_{d,ex}$$

b)

$$\pi_{d,in}=0.97$$

$$\pi_d=\pi_{d,ex}\pi_{d,in}=0.8952\cdot0.97=0.868$$

c)

$$\tau_r=\frac{T_{t0}}{T_0}=1+\frac{\gamma-1}{2}M_0^2\rightarrow\tau_r=1+0.2\cdot(1.6)^2=1.512$$

$$\eta_d=\frac{\tau_r(\pi_d)^{\gamma-1/\gamma}-1}{\tau_r-1}\rightarrow\eta_d=\frac{1.512(0.868)^{1.4-1/1.4}-1}{1.512-1}=0.883$$

Problem 2 A compressor is composed of 15 stages, each with a pressure ratio of 1.24 and stage efficiency of 0.92.

a) What is the compressor pressure ratio?
b) What is the compressor efficiency?
c) What is the polytropic efficiency?
d) Evaluate the compressor efficiency assuming that the polytropic efficiency is equal to the stage efficiency.

Problem 2 Solution

(a)

$$\pi_s = 1.24$$

$$\pi_c = \pi_s^N \Rightarrow \pi_c = (1.24)^{15} = 25.196$$

(b)

$$\eta_s = 0.92$$

$$\eta_c = \frac{(\pi_c)^{(\gamma-1)/\gamma} - 1}{(\pi_c)^{(\gamma-1)/(\gamma\eta_s)} - 1} = 0.8783$$

(c)

$$e_c = \frac{[(\gamma-1)/\gamma]\, ln\pi_s}{ln\left[1 + (1/\eta_s)((\pi_s)^{(\gamma-1)/\gamma} - 1)\right]} = 0.9224$$

(d)

$$\eta_c = \frac{(pi_c)^{(\gamma-1)/\gamma} - 1}{(pi_c)^{(\gamma-1)/(\gamma e_c)} - 1}$$

if $e_c = \eta_c$ then by either of the two above equations, we get the value for η_c.